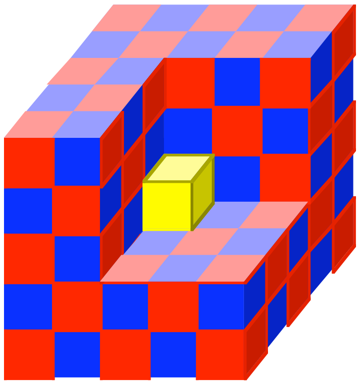
Photonic Crystals:

Periodic Surprises in Electromagnetism

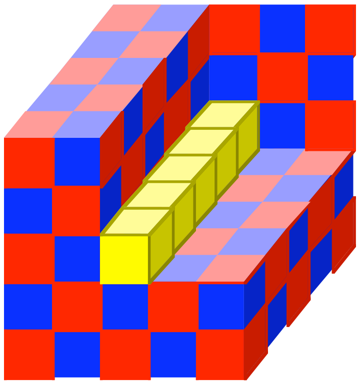
Steven G. Johnson

MIT

A “Defective” Lecture

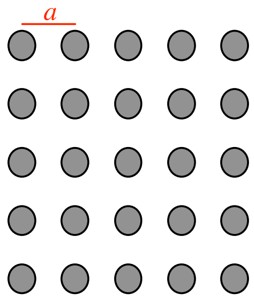


cavity



waveguide

The Story So Far…



Waves in periodic media can have:

- propagation with no scattering (conserved *k*)
- photonic band gaps (with proper ε function)

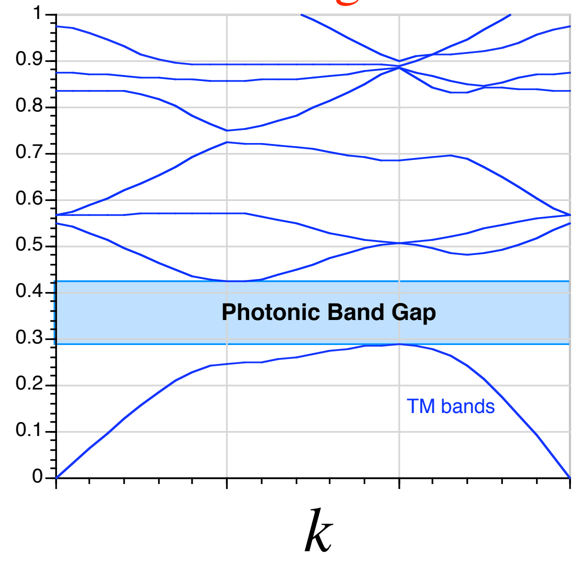
Eigenproblem gives simple insight:

Bloch form: $\vec{H} = e^{i(\vec{k}\cdot\vec{x} - \omega t)} \vec{H}_{\vec{k}}(\vec{x}) \quad \Rightarrow$

$$\left[\underbrace{(\nabla + i\vec{k}) \times \frac{1}{\varepsilon}(\nabla + i\vec{k}) \times}_{\hat{\Theta}_{\vec{k}}} \right] \vec{H}_{\vec{k}} = \left(\frac{\omega_n(\vec{k})}{c} \right)^2 \vec{H}_{\vec{k}}$$

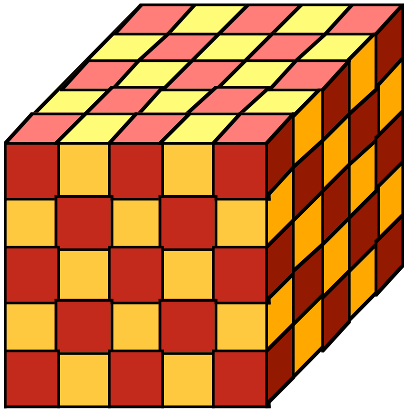
Hermitian –> complete, orthogonal, variational theorem, *etc.*

band diagram

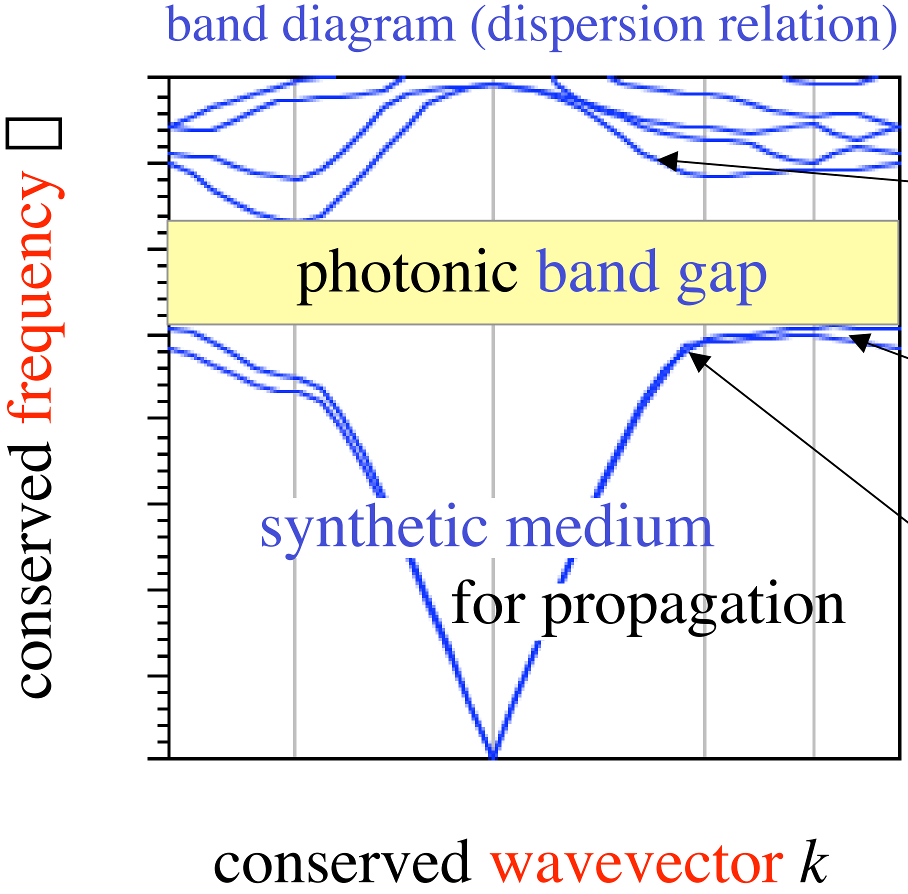


Properties of Bulk Crystals

by Bloch's theorem



(cartoon)



backwards slope:
negative refraction

$d\omega/dk \to 0$: slow light
(e.g. DFB lasers)

strong curvature:
super-prisms, …
(+ negative refraction)

Applications of Bulk Crystals

using near-band-edge effects

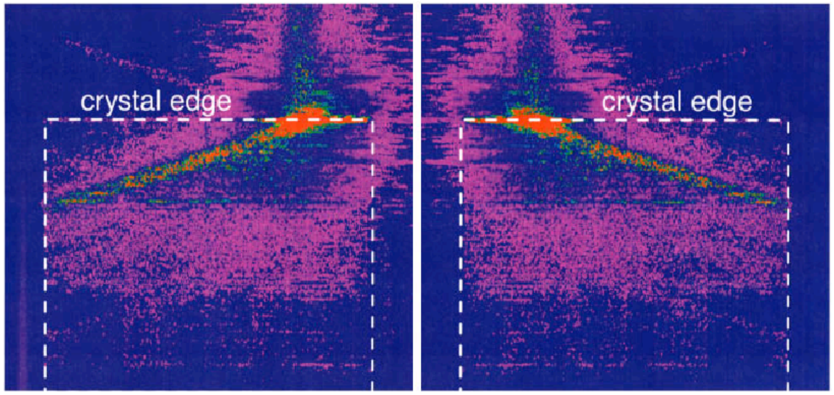
Zero group-velocity $d\omega/dk$: distributed feedback (DFB) lasers

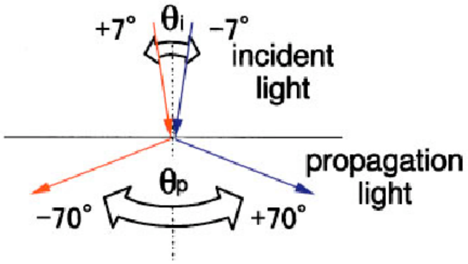
divergent dispersion

(*i.e.* curvature):

Superprisms

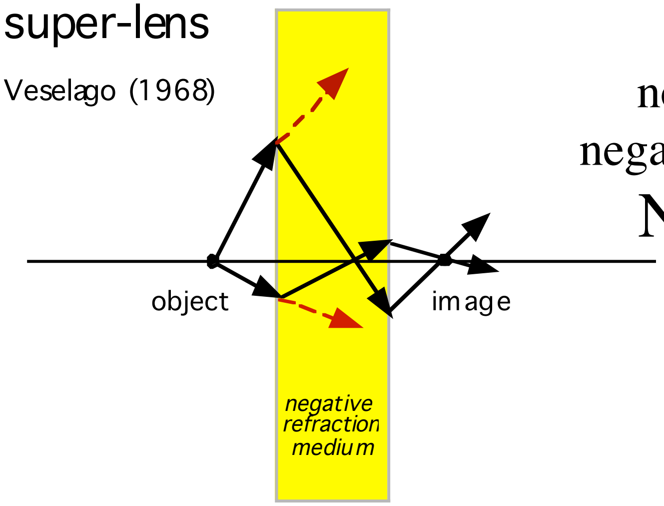
[Kosaka, *PRB* **58**, R10096 (1998).]





super-lens

Veselago (1968)

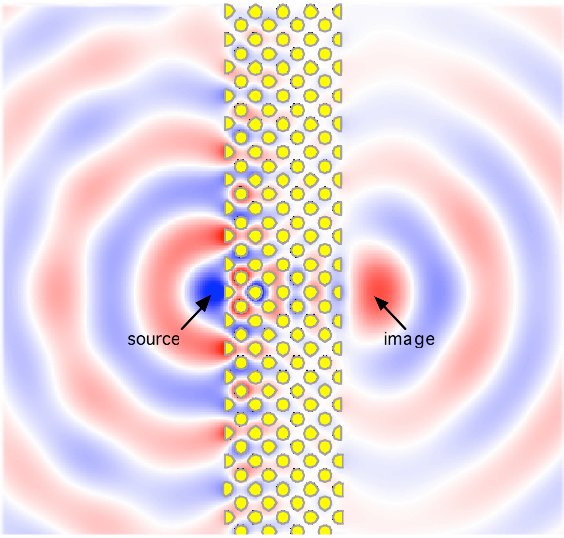


negative group-velocity **or**
negative curvature ("eff. mass"):

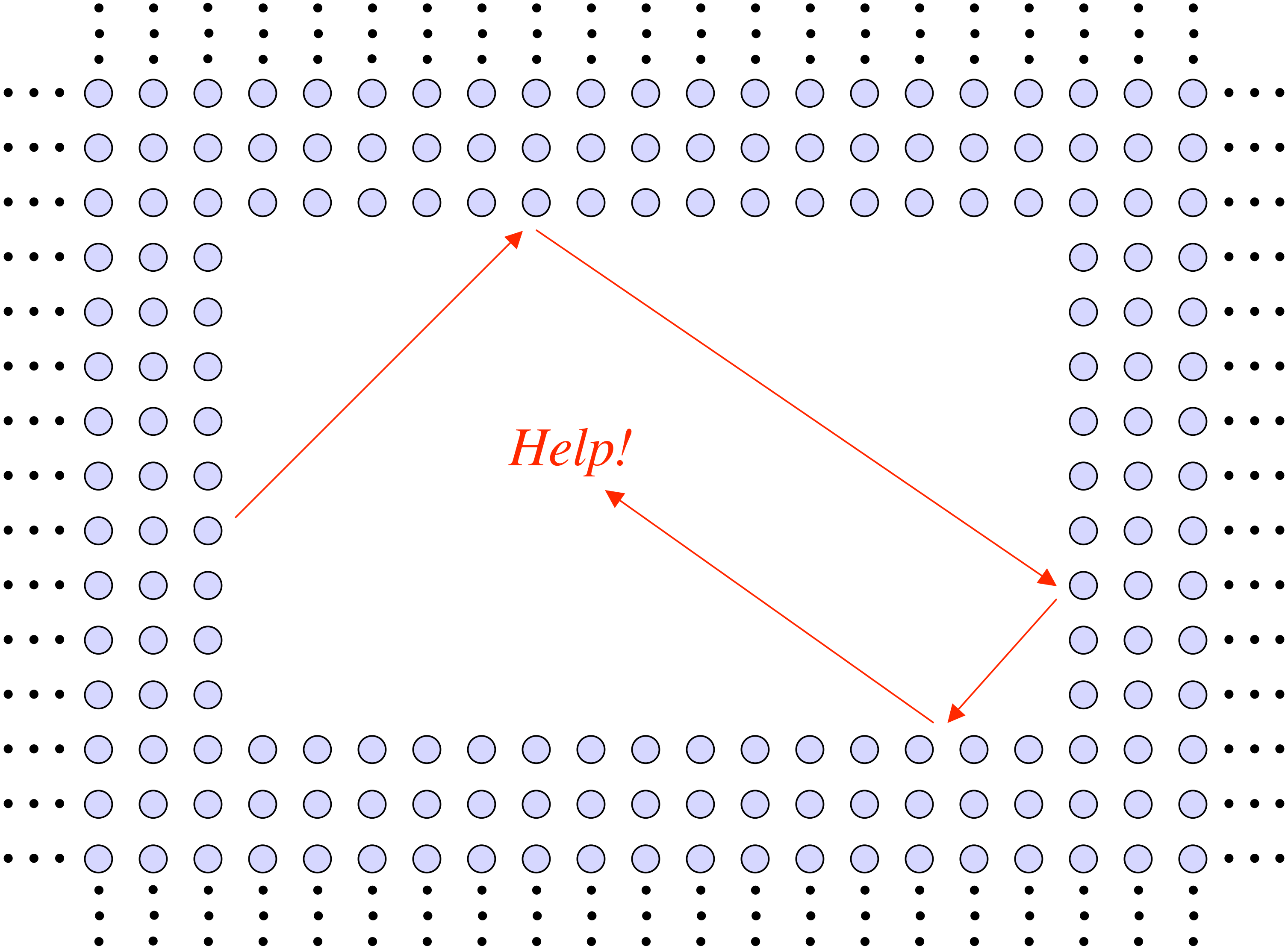
Negative refraction,

Super-lensing

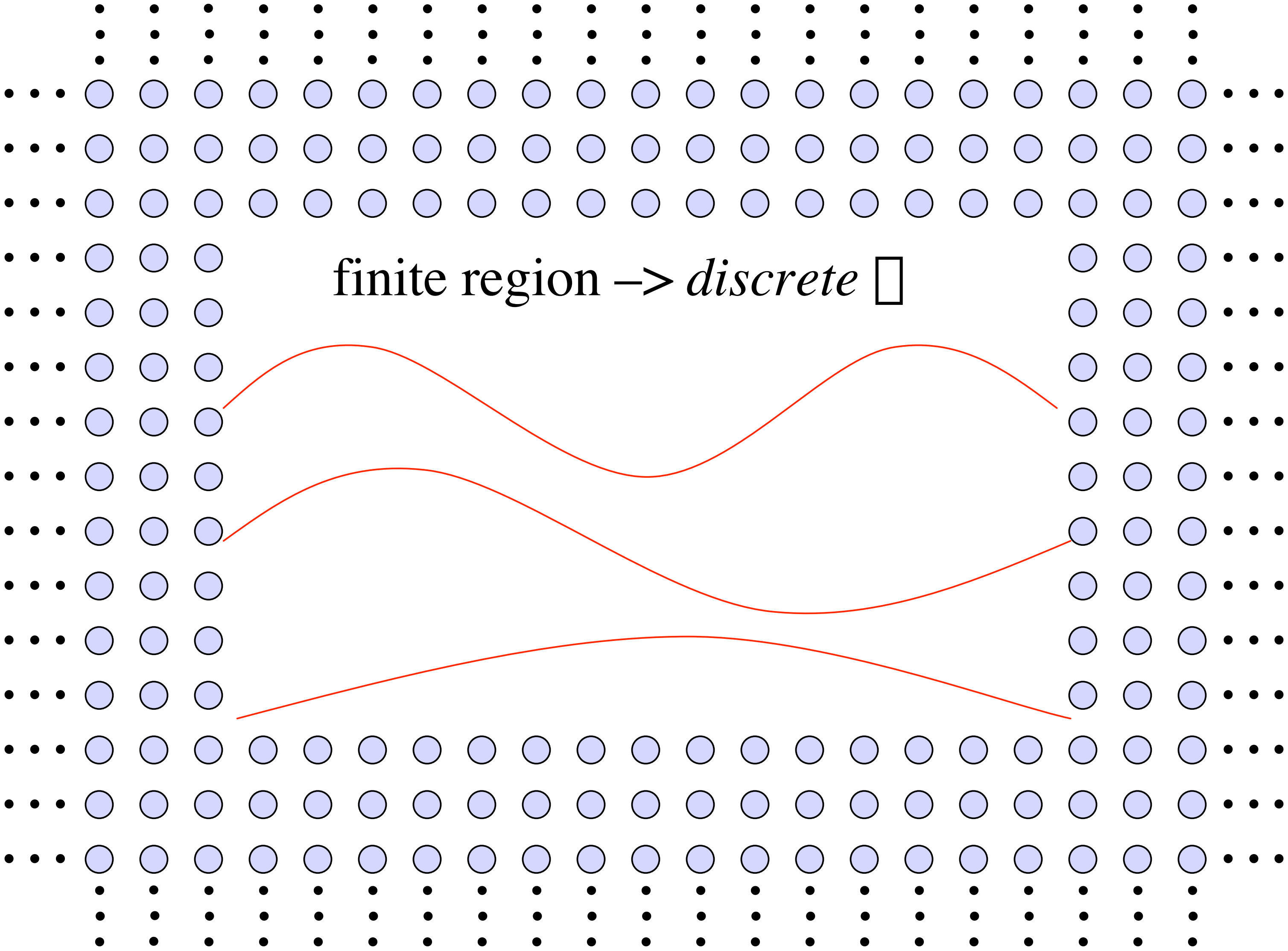
[C. Luo *et al.*,
Appl. Phys. Lett. **81**, 2352 (2002)]



Cavity Modes

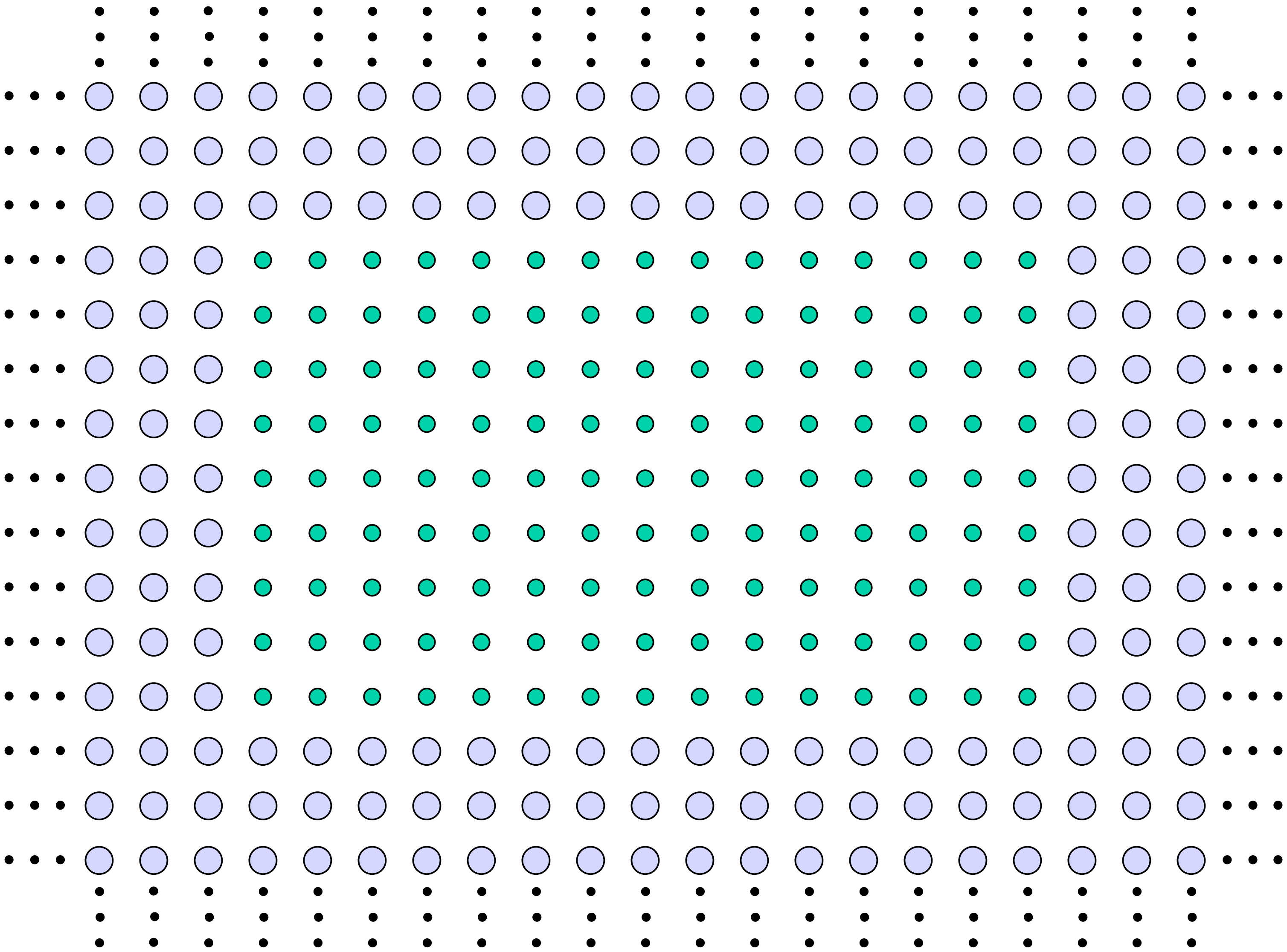


Cavity Modes

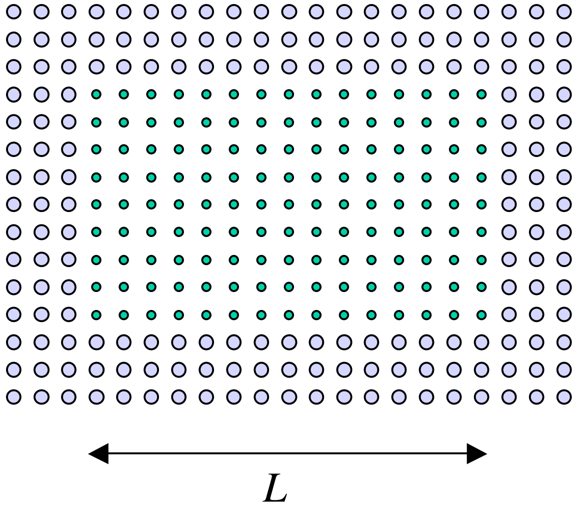


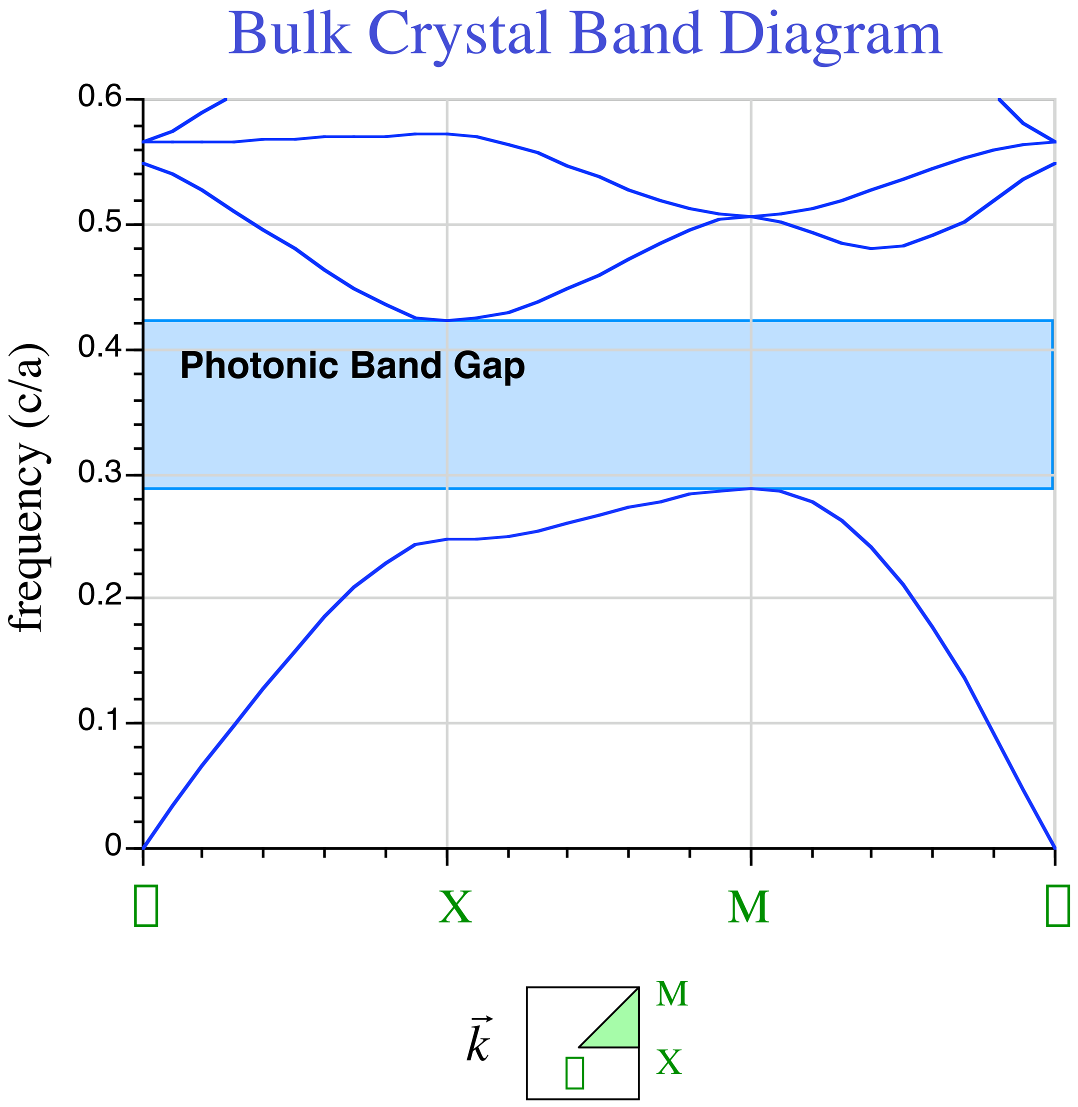
finite region –> *discrete* □

Cavity Modes: Smaller Change

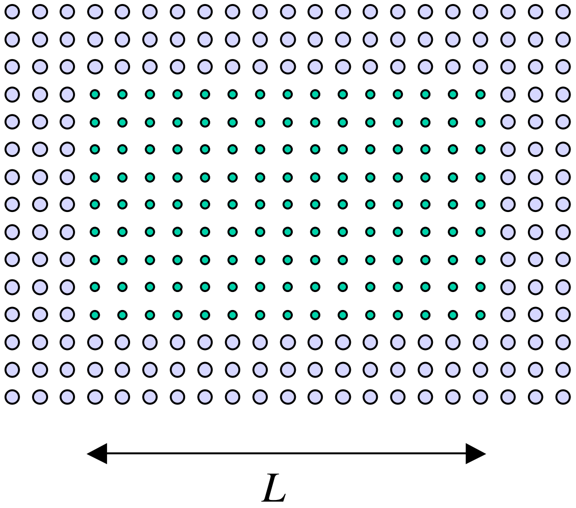


Cavity Modes: Smaller Change





Cavity Modes: Smaller Change

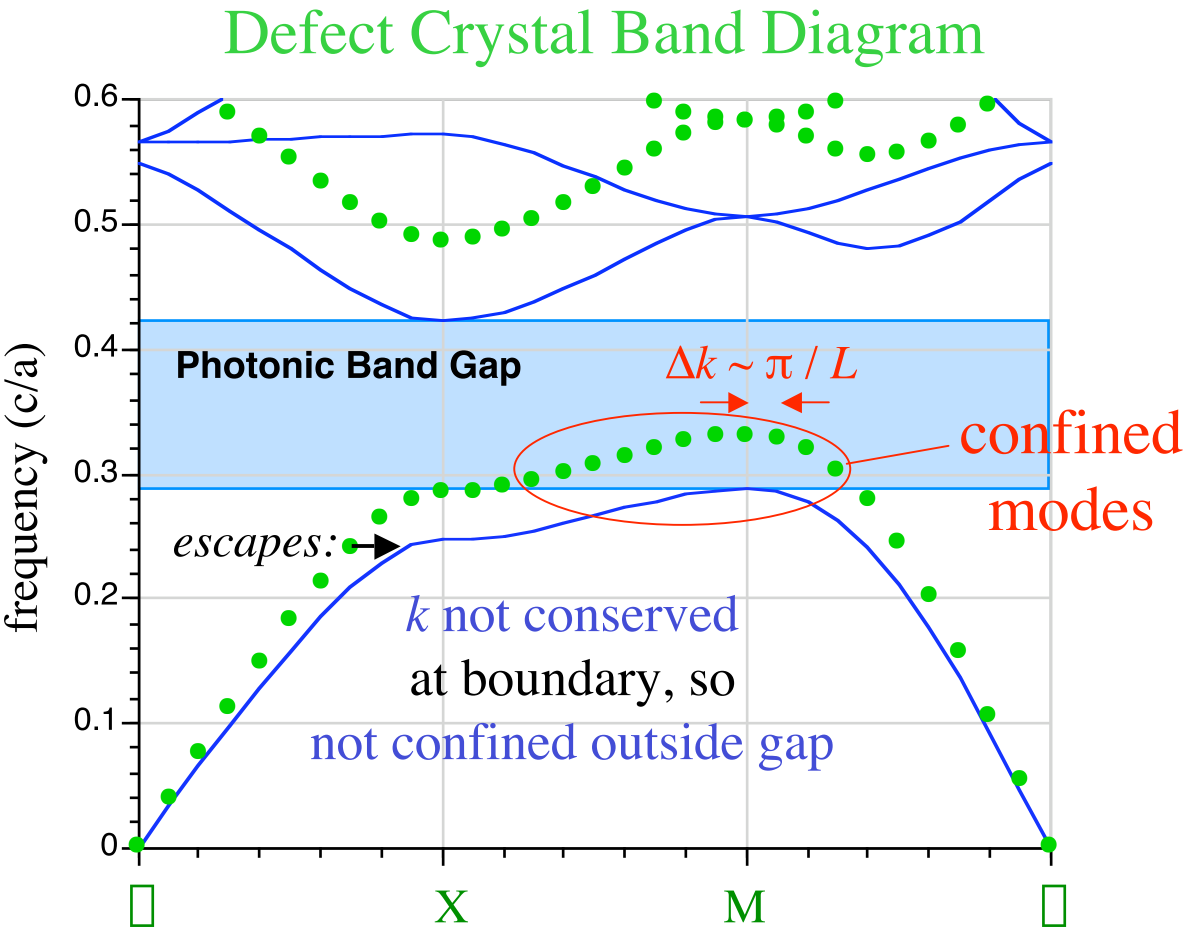


Defect bands are shifted *up* (less ε)

with *discrete k*

$$\# \cdot \frac{\lambda}{2} \sim L \qquad (k \sim 2\pi / \lambda)$$

Defect Crystal Band Diagram

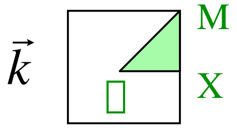


Δ*k* ~ π / *L*

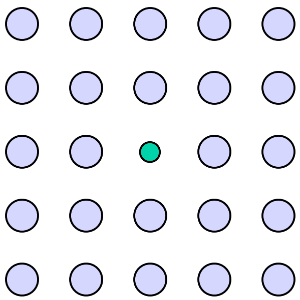
confined modes

escapes:

k not conserved at boundary, so not confined outside gap



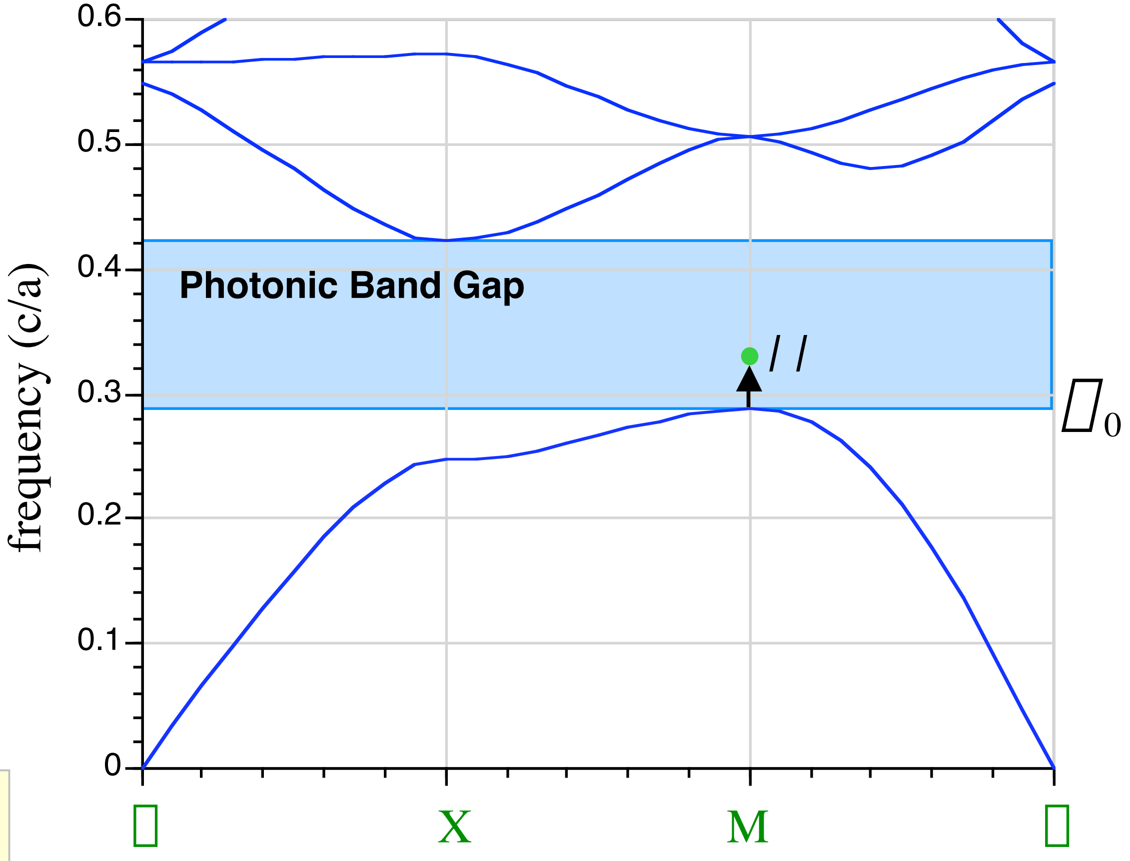
Single-Mode Cavity

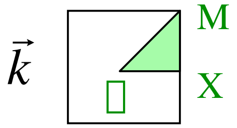


A *point defect*
can push up
a single mode
from the band edge

field decay ~ $\sqrt{\frac{\square\ \square\square_0}{\text{curvature}}}$

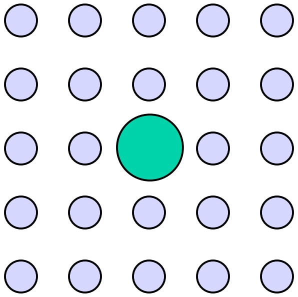
Bulk Crystal Band Diagram





(k not conserved)

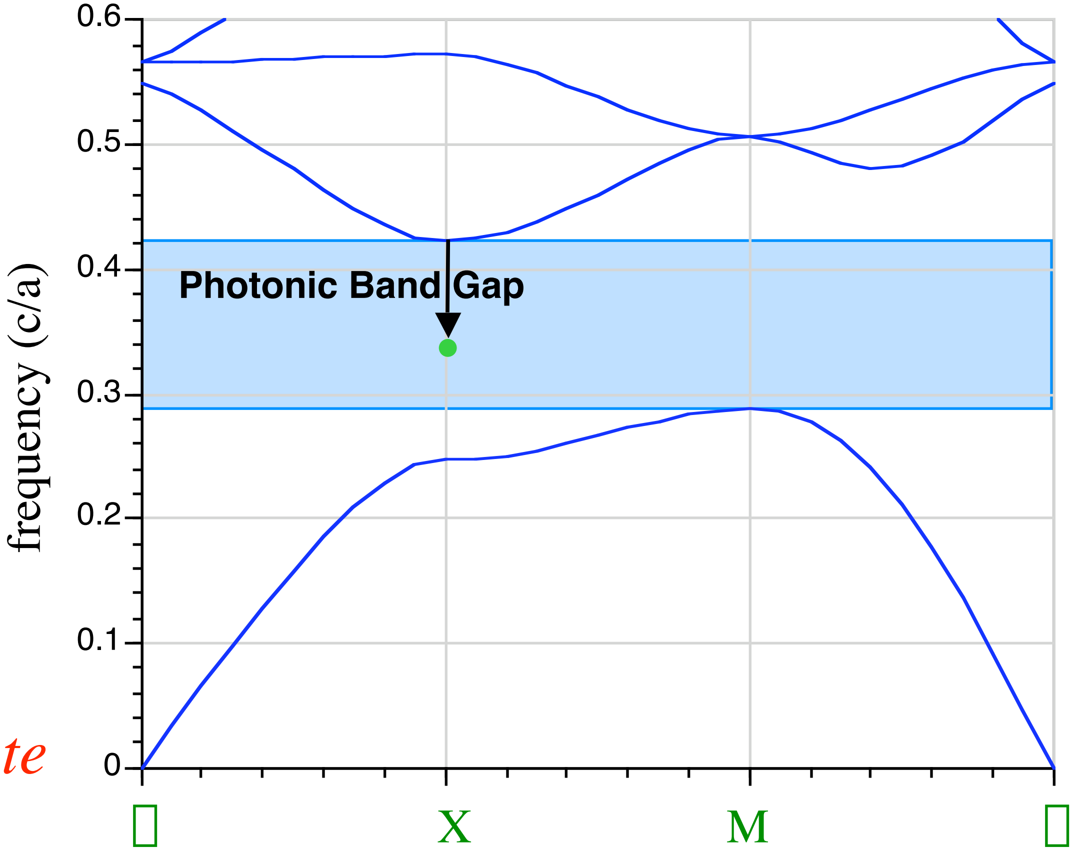
“Single”-Mode Cavity

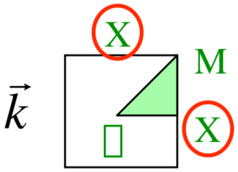


A *point defect*
can pull down
a “single” mode

…here, doubly-*degenerate*
(*two* states at *same* □)

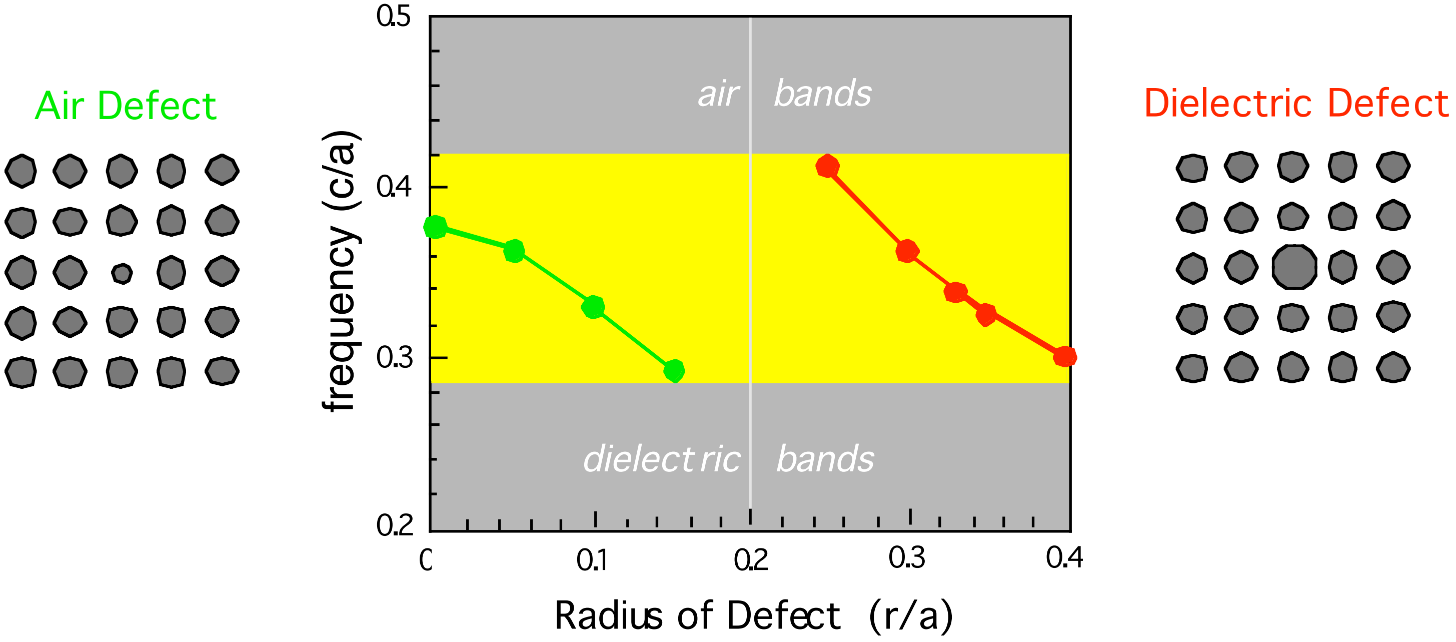
Bulk Crystal Band Diagram

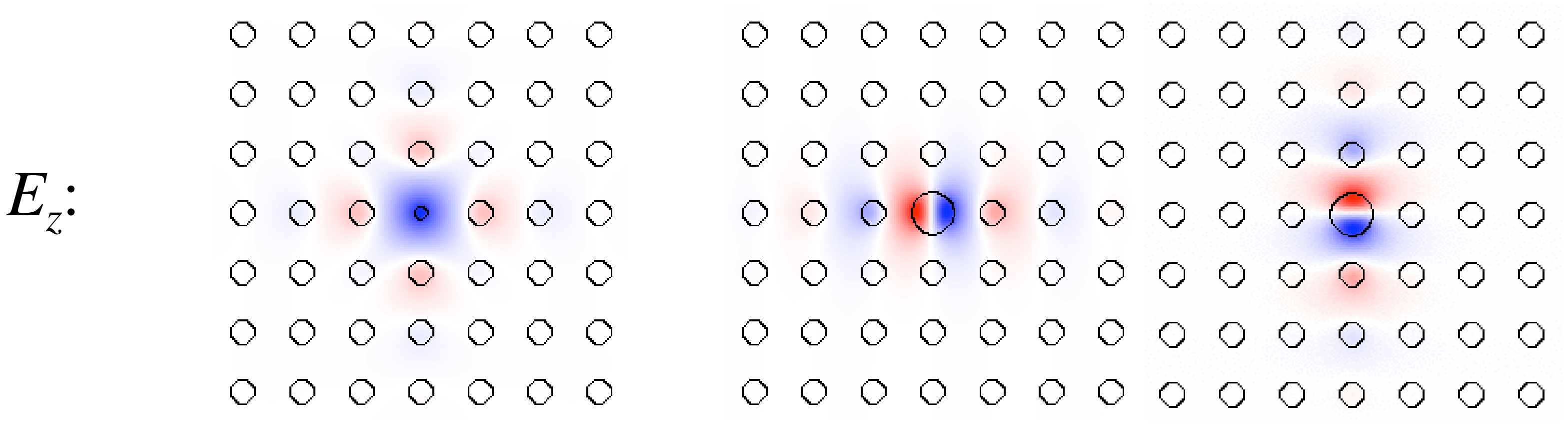




(k not conserved)

Tunable Cavity Modes

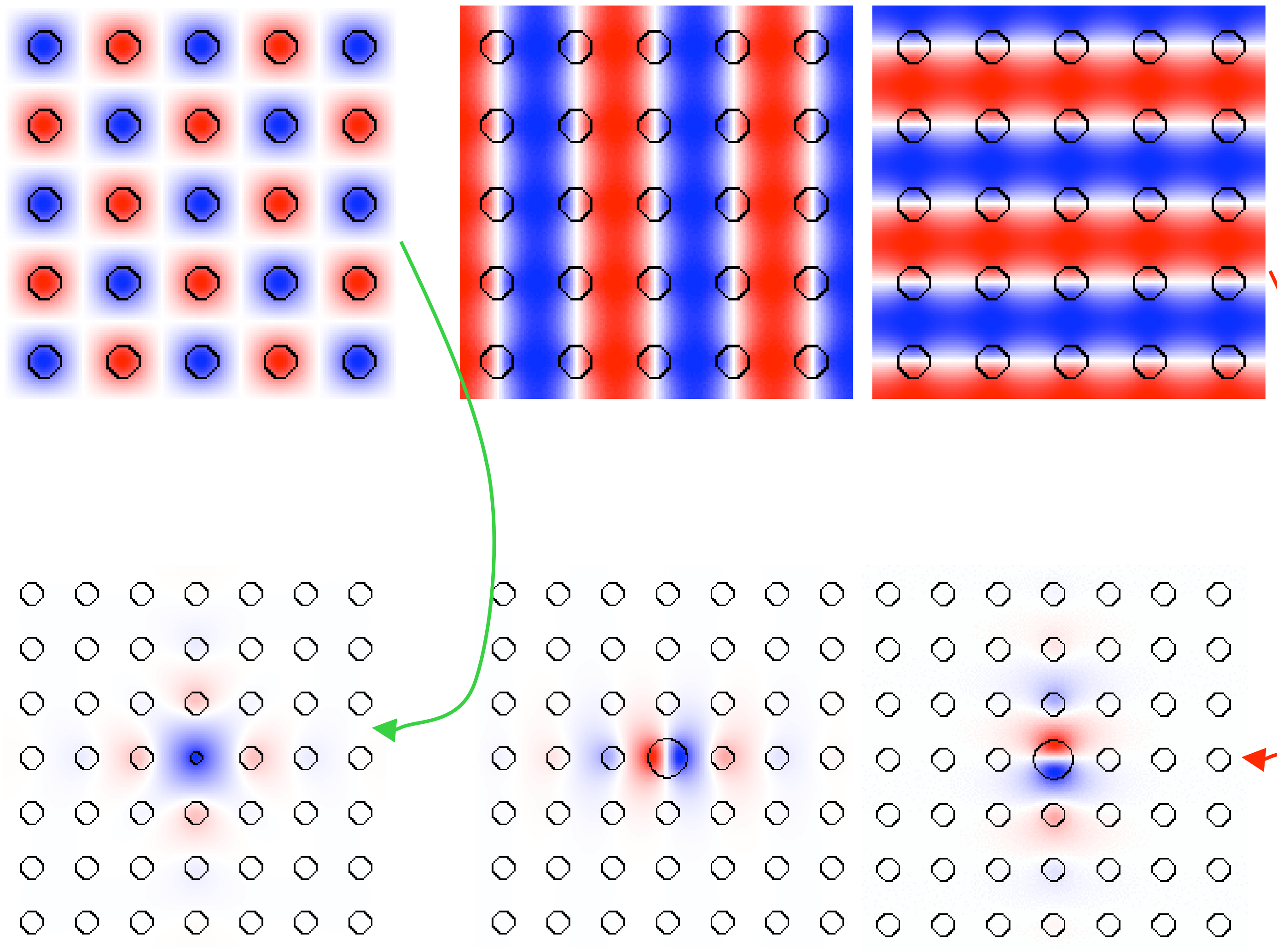




monopole

dipole

Tunable Cavity Modes



band #1 at M

band #2 at X's

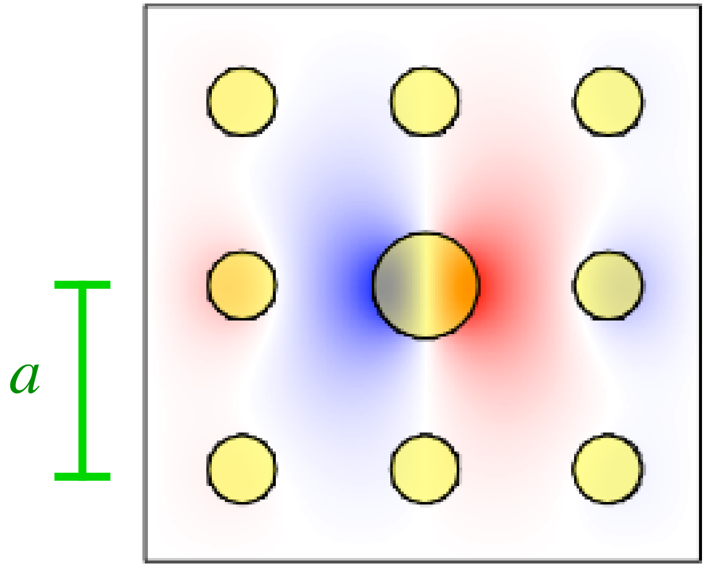
multiply by exponential decay

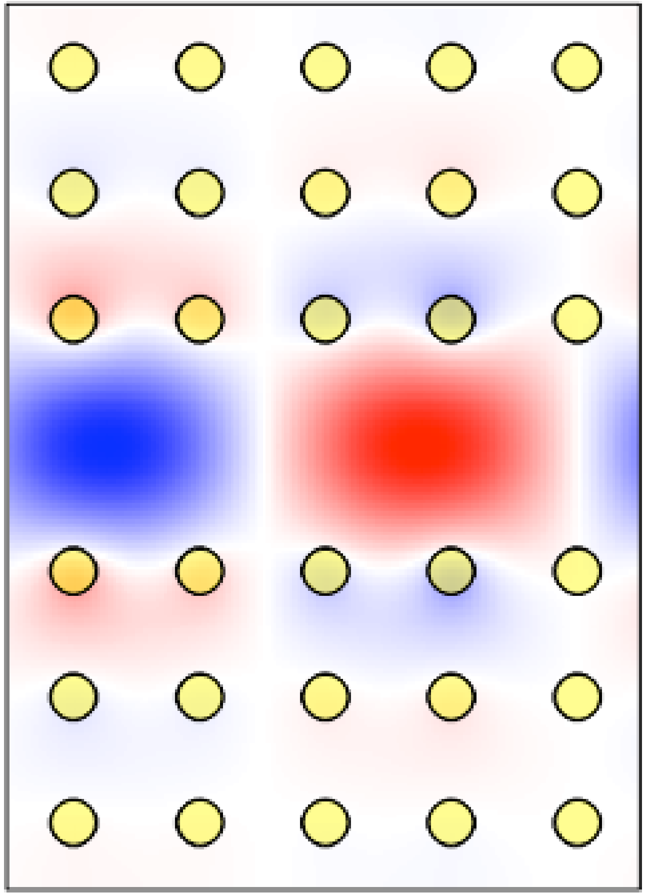
E_z:

monopole

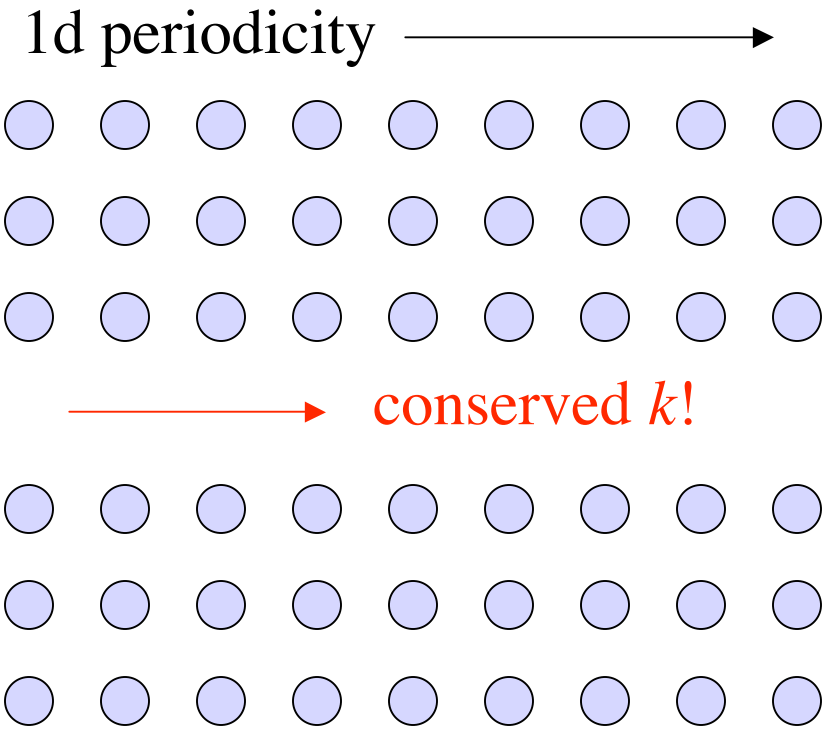
dipole

Defect Flavors

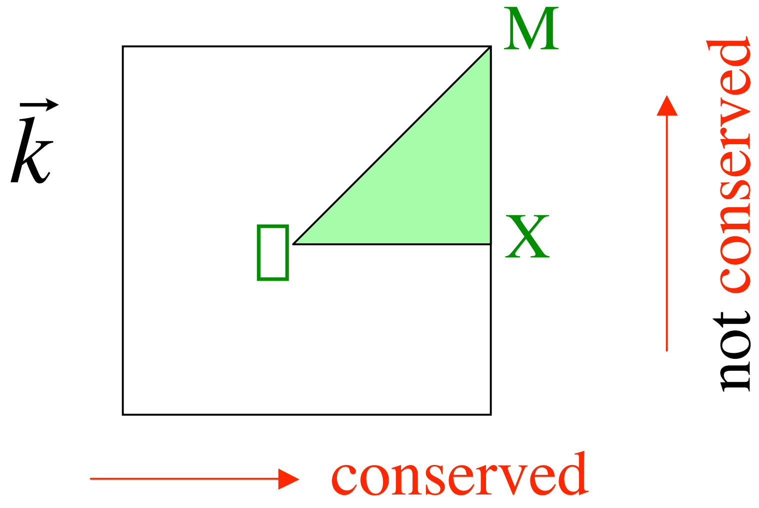

microcavities


waveguides

Projected Band Diagrams


1d periodicity

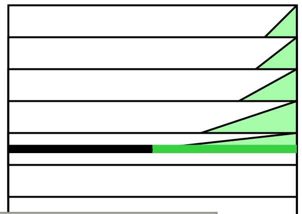
conserved *k*!



not conserved

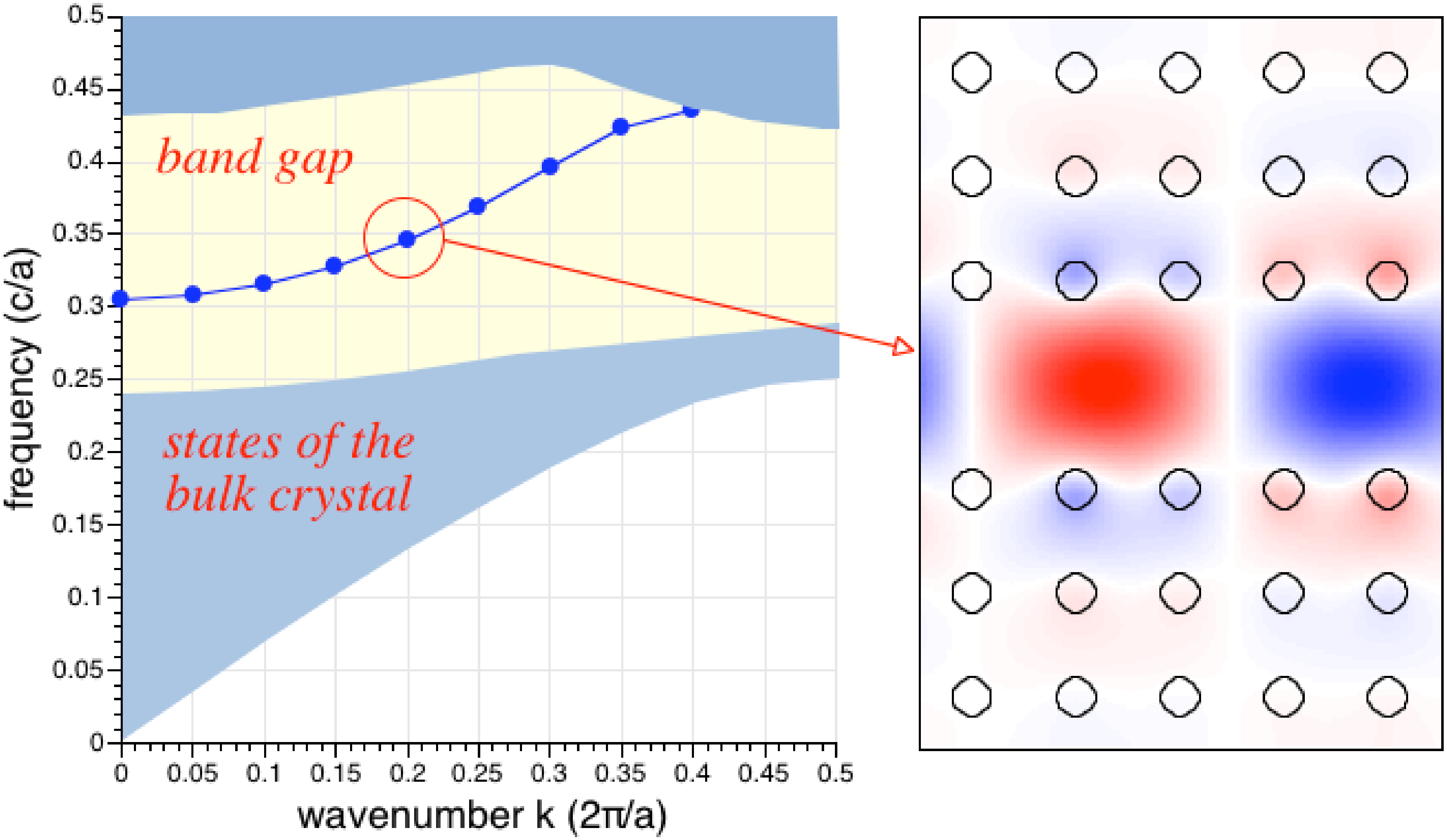
conserved

So, plot □ vs. k_x only…project Brillouin zone onto □–X:



gives continuum of bulk states + discrete guided band(s)

Air-waveguide Band Diagram



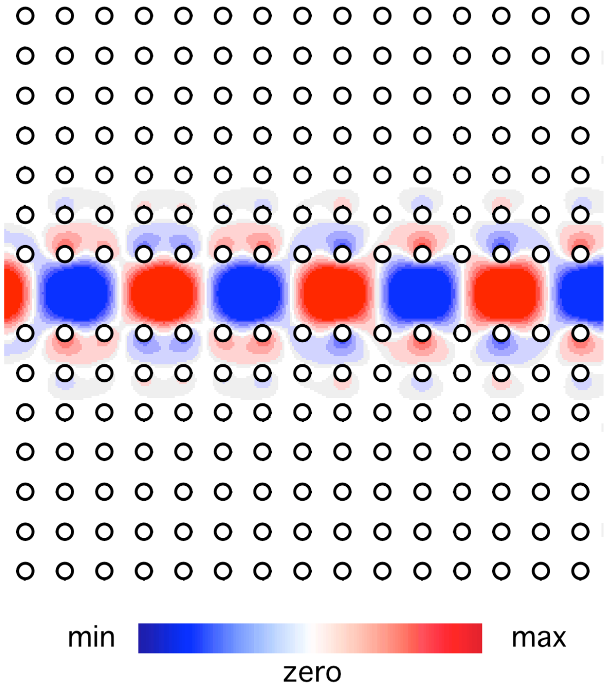
any state in the gap cannot couple to bulk crystal –> localized

(Waveguides don't really need a *complete* gap)

Fabry-Perot waveguide:

We'll exploit this later, with photonic-crystal fiber…

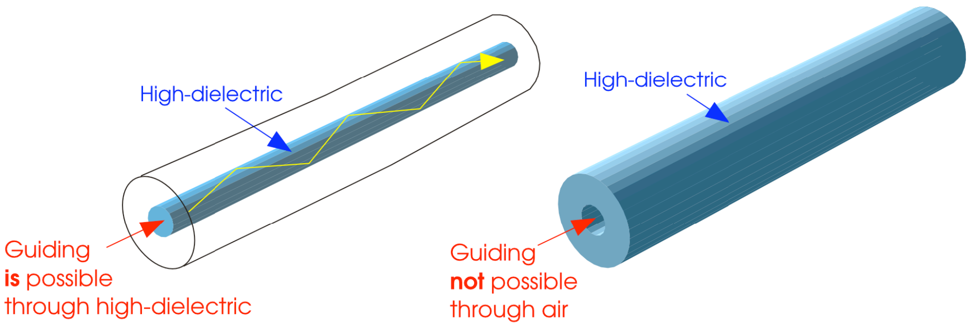
So What?



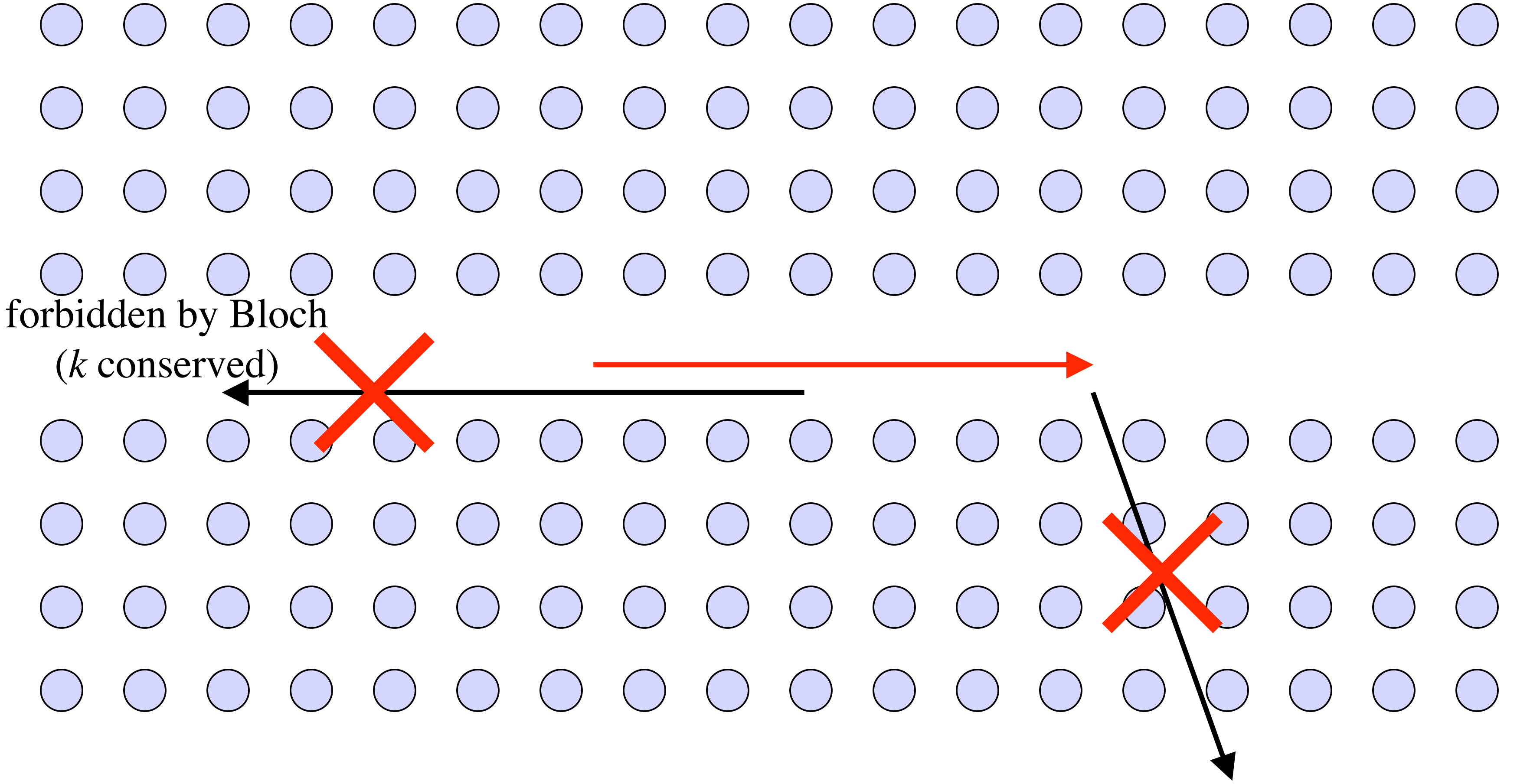
Guiding Optical Light through Air

- Reduction of Absorption Losses
- Reduction of Non-Linearity Effects
- High Power Transmission

Total Internal Reflection



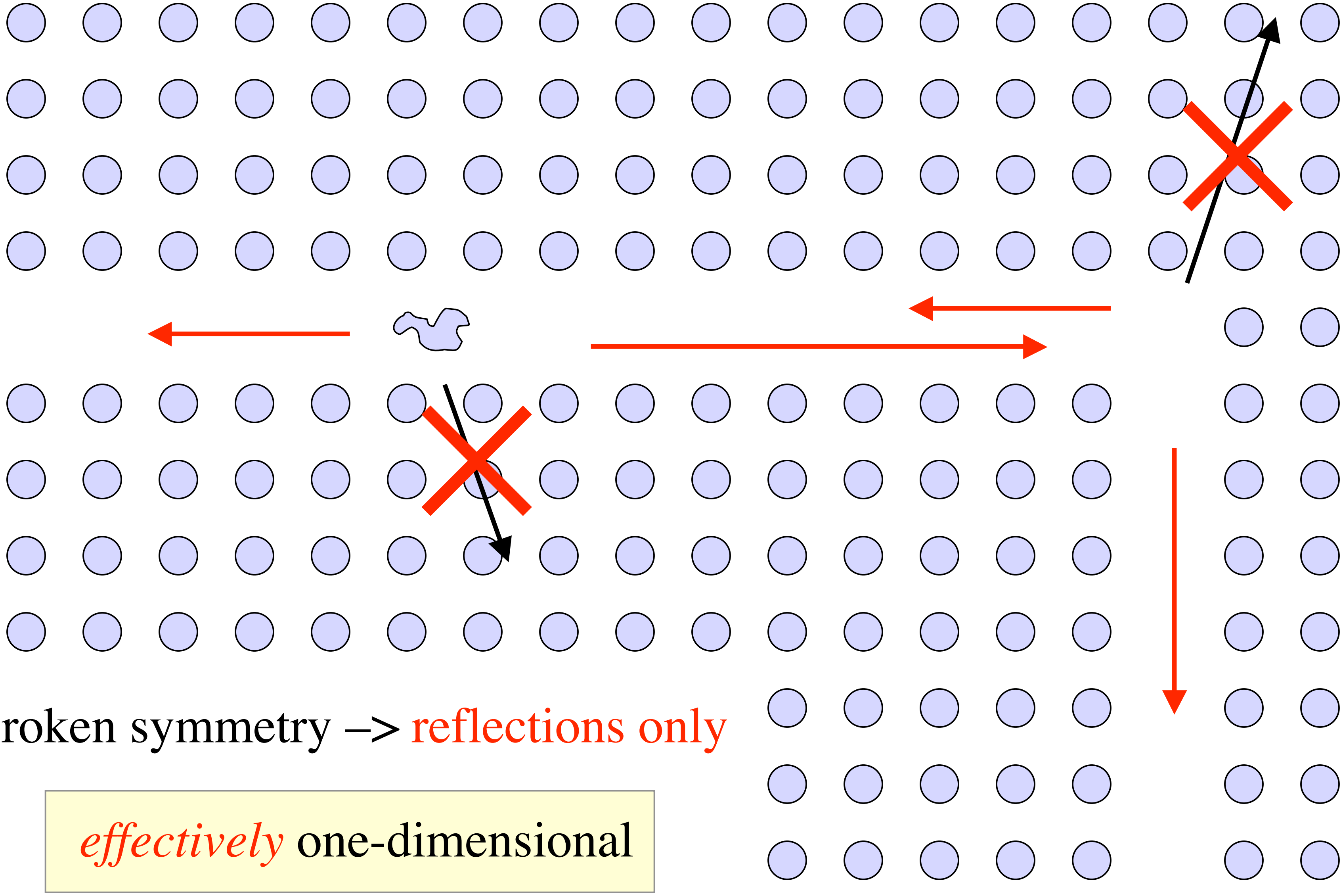
Review: Why no scattering?



forbidden by Bloch
(k conserved)

forbidden by gap
(except for finite-crystal tunneling)

Benefits of a complete gap…

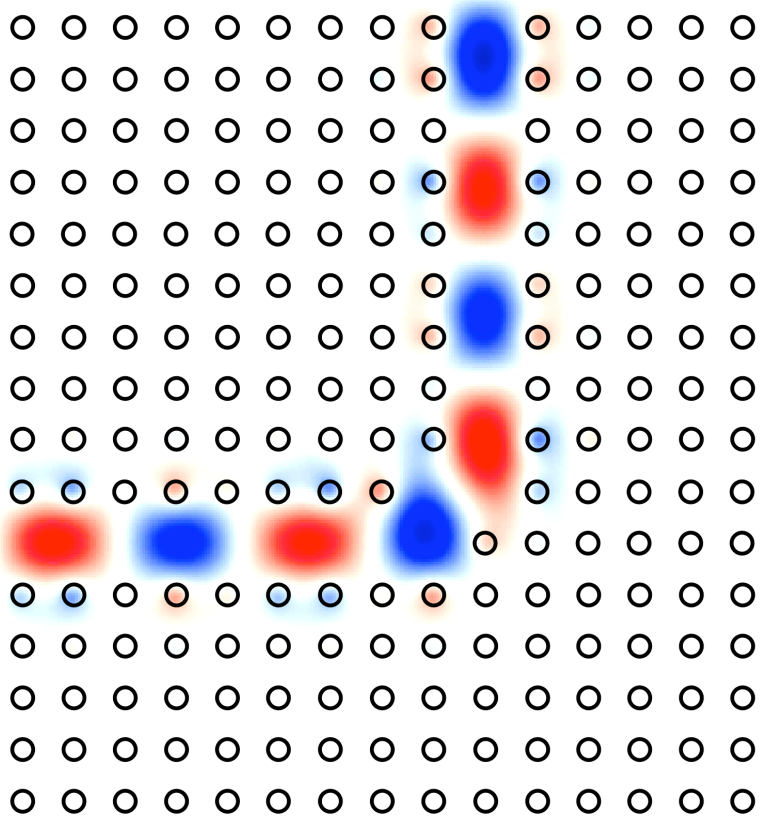


broken symmetry –> reflections only

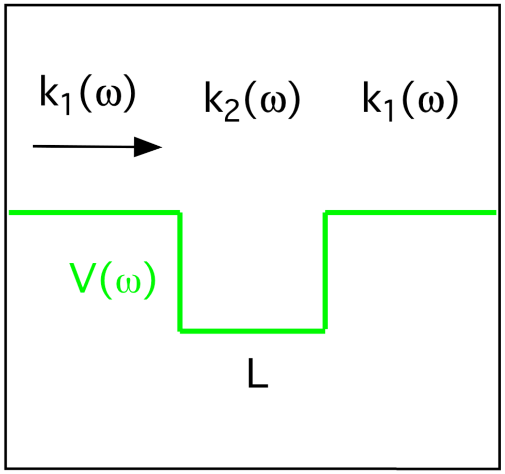
effectively one-dimensional

Lossless Bends

100% Transmission through Sharp Bends



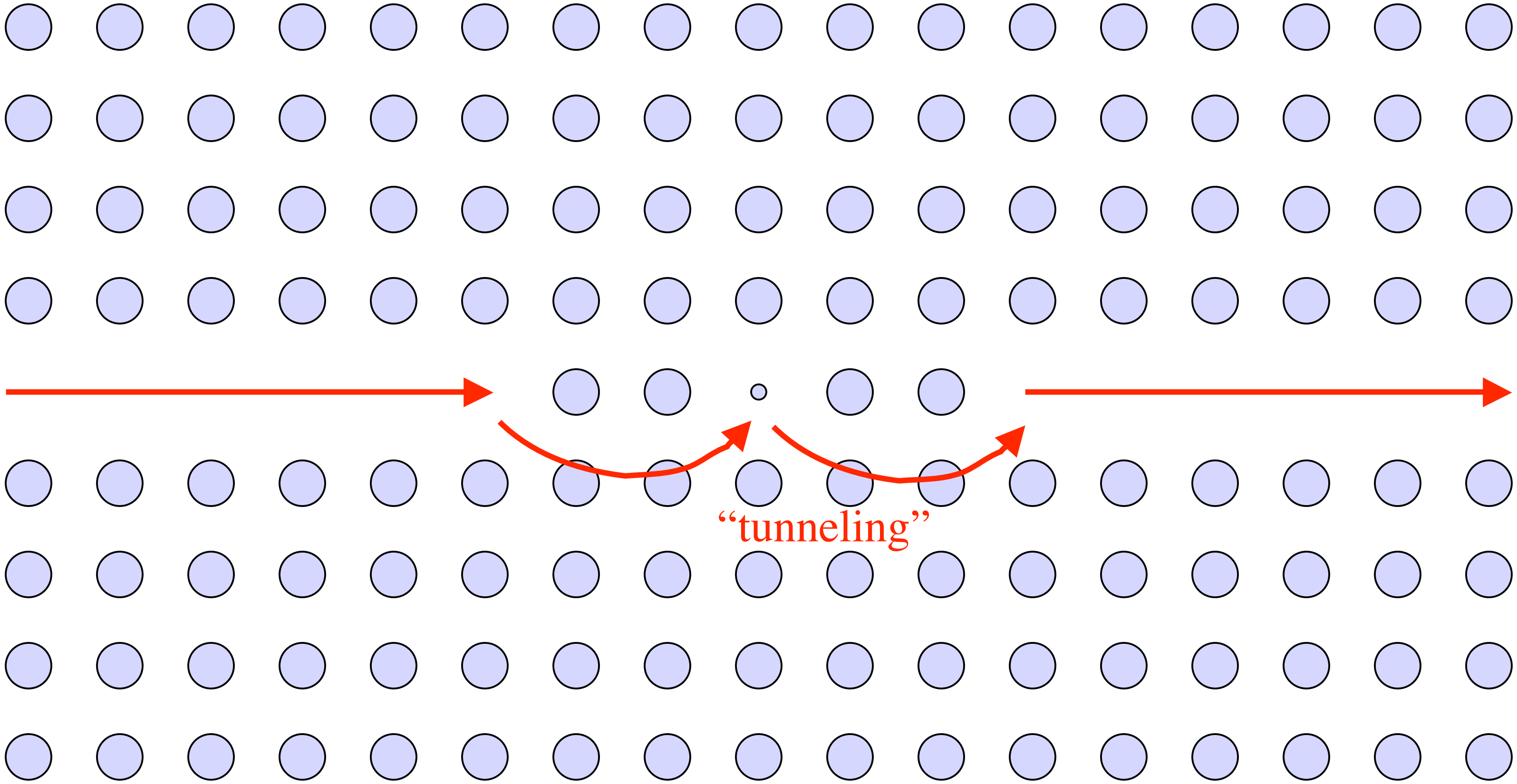
Maps onto problem of Electron Resonant Scattering in 1D



[A. Mekis *et al.*,
Phys. Rev. Lett. **77**, 3787 (1996)]

symmetry + single-mode + "1d" = resonances of 100% transmission

Waveguides + Cavities = Devices



Ugh, must we simulate this to get the basic behavior?

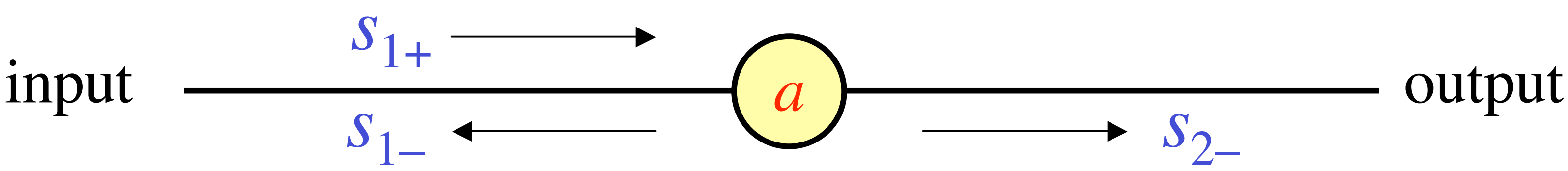
No! Use "coupling-of-modes-in-time" (coupled-mode theory)…

[H. Haus, *Waves and Fields in Optoelectronics*]

"Coupling-of-Modes-in-Time"

(a form of coupled-mode theory)

[H. Haus, *Waves and Fields in Optoelectronics*]



resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = flux

$|a|^2$ = energy

$$\frac{da}{dt} = -i\omega_0 a - \frac{2}{\tau} a + \sqrt{\frac{2}{\tau}} s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}} a, \quad s_{2-} = \sqrt{\frac{2}{\tau}} a$$

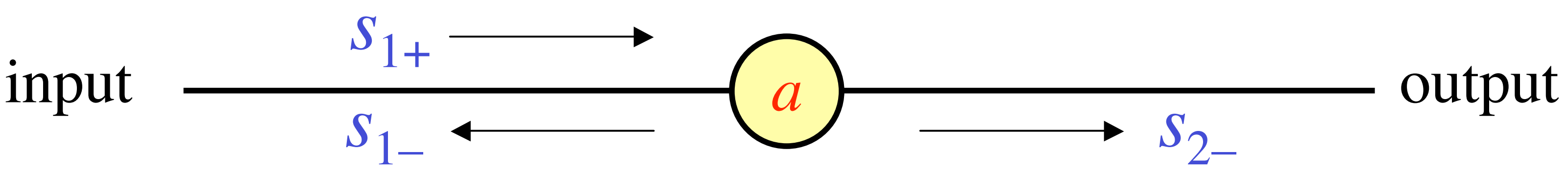
assumes only:

- exponential decay
  (strong confinement)
- conservation of energy
- time-reversal symmetry

“Coupling-of-Modes-in-Time”

(a form of coupled-mode theory)

[H. Haus, *Waves and Fields in Optoelectronics*]

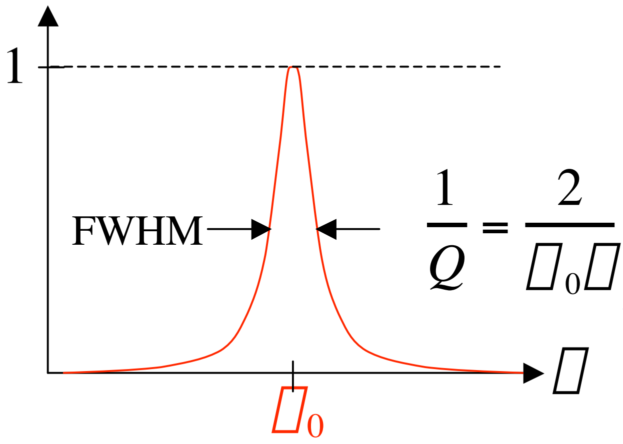


resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = flux

$|a|^2$ = energy

transmission T
$= |s_{2-}|^2 / |s_{1+}|^2$



$$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$$

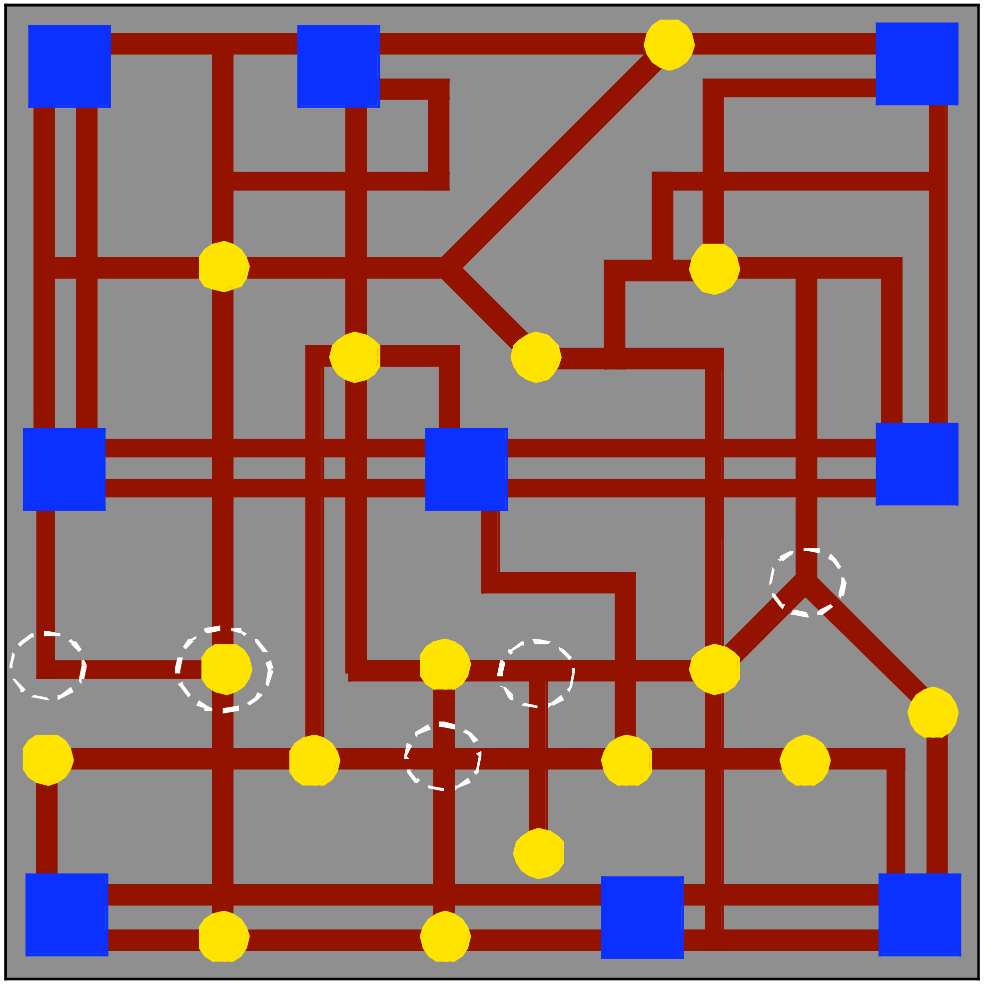
…quality factor Q

T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

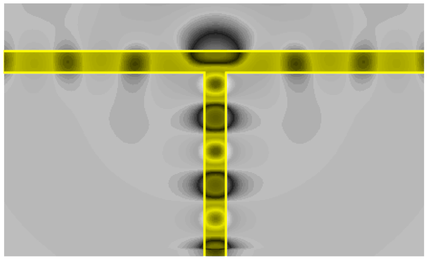
A Menagerie of Devices

1.55 microns

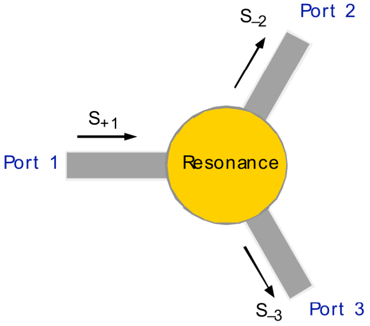


Wide-angle Splitters

only 30%

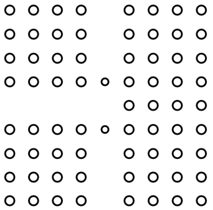


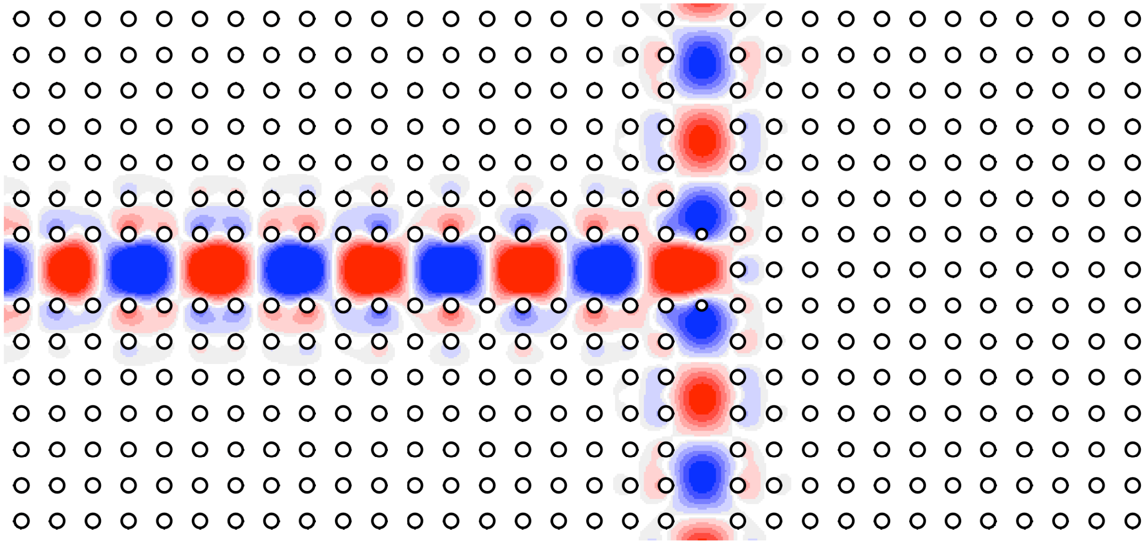
C. Manolatou



100% Transmission

$$1/\tau_1 = 1/\tau_2 + 1/\tau_3$$

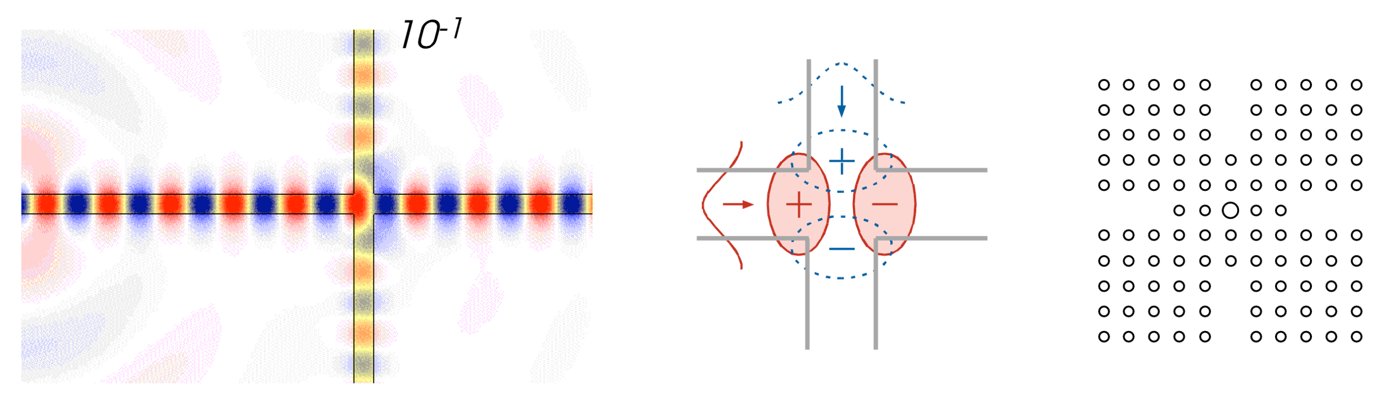


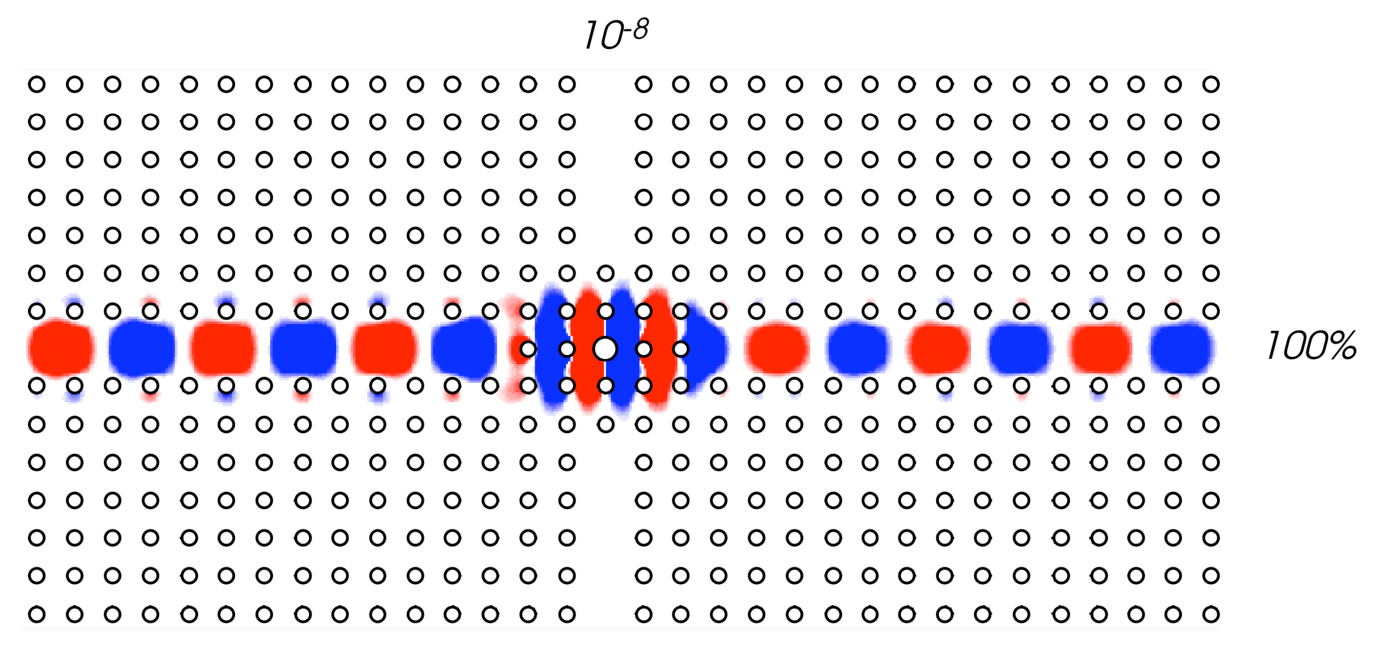


100%

[S. Fan *et al.*, *J. Opt. Soc. Am. B* **18**, 162 (2001)]

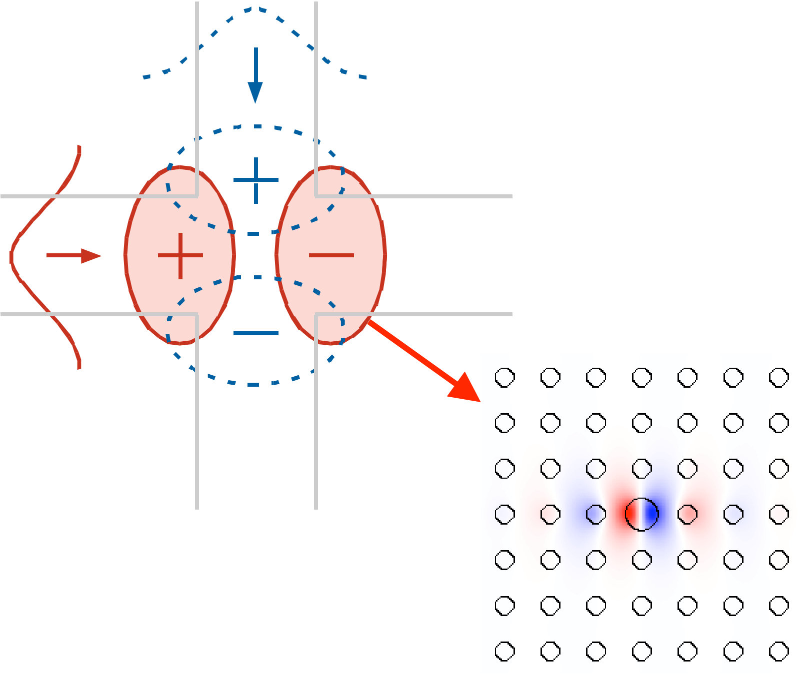
Waveguide Crossings

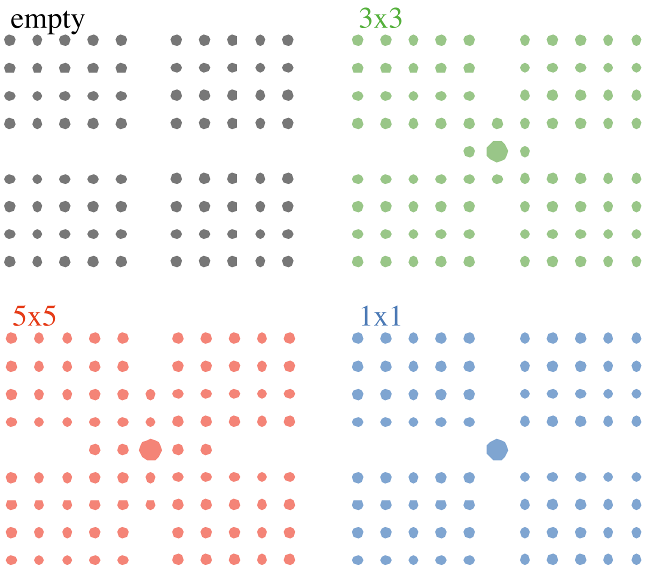


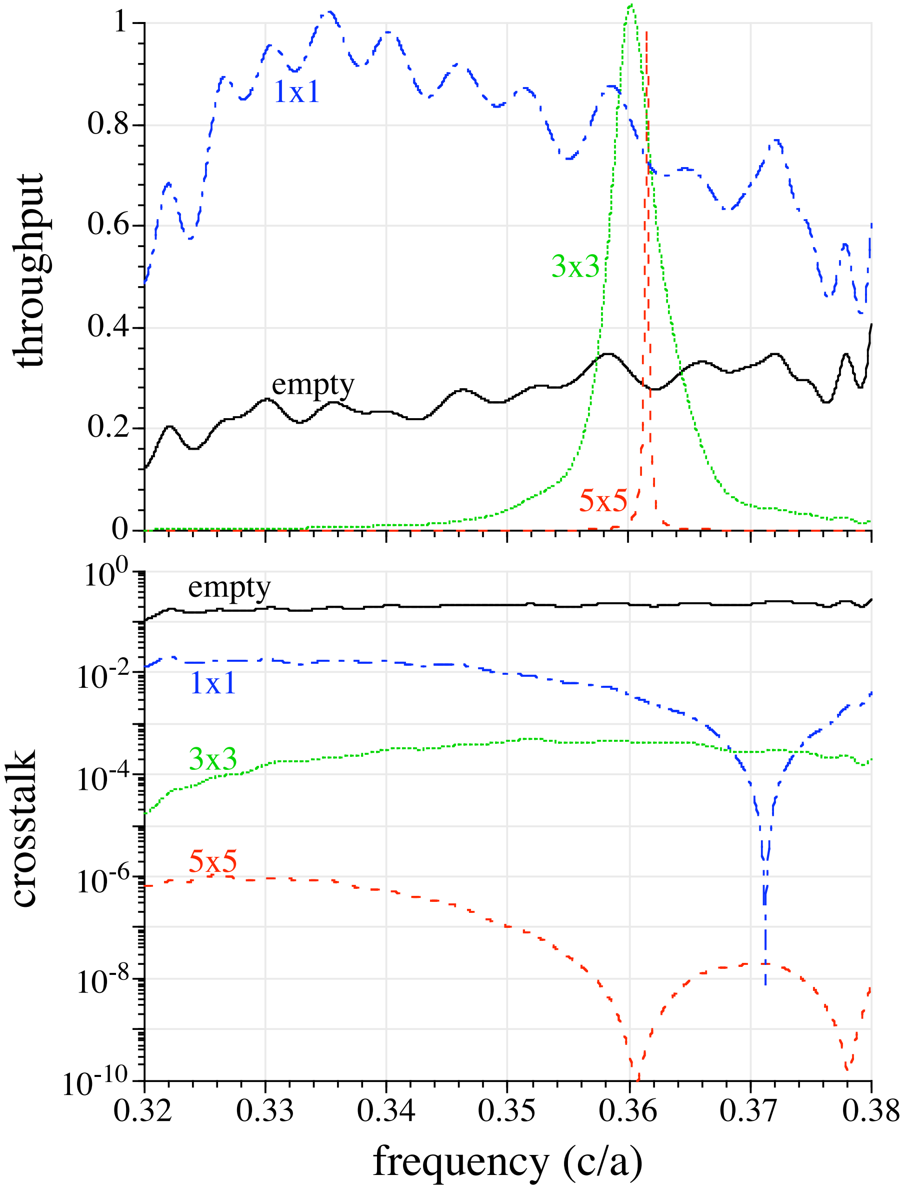


[S. G. Johnson *et al.*, *Opt. Lett*. **23**, 1855 (1998)]

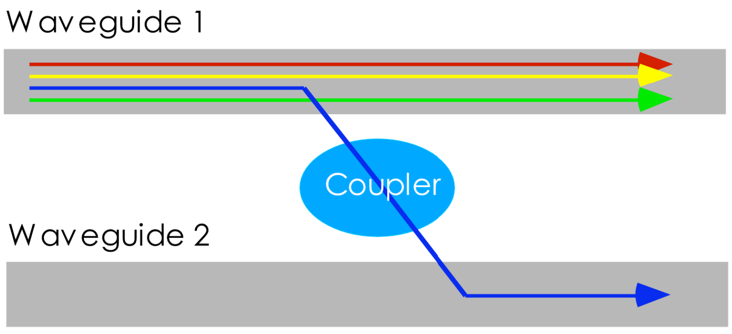
Waveguide Crossings







Channel-Drop Filters

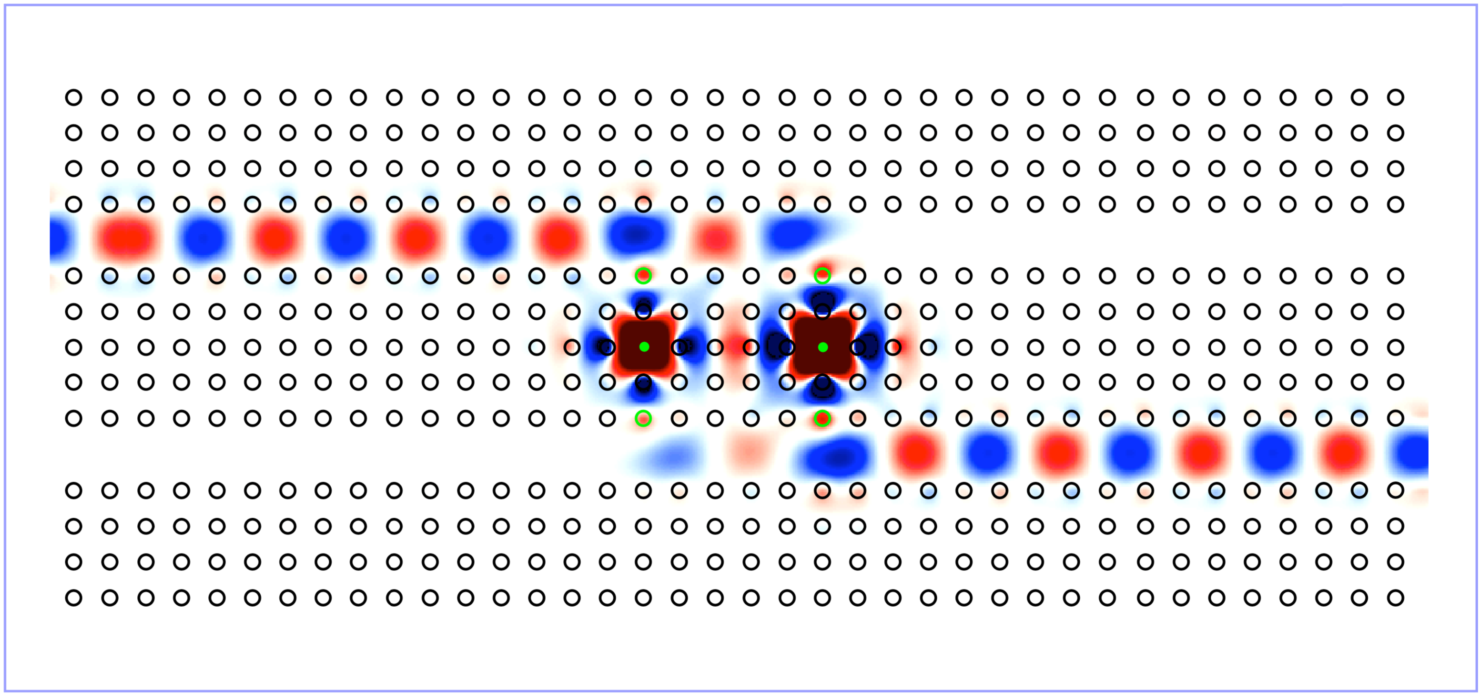


Criteria for Perfect Channel-dropping

Two resonant modes with even and odd symmetry

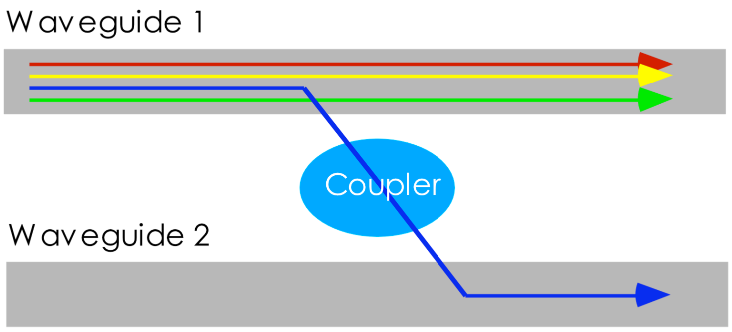
Modes must be degenerate

Modes must have equal decay rates



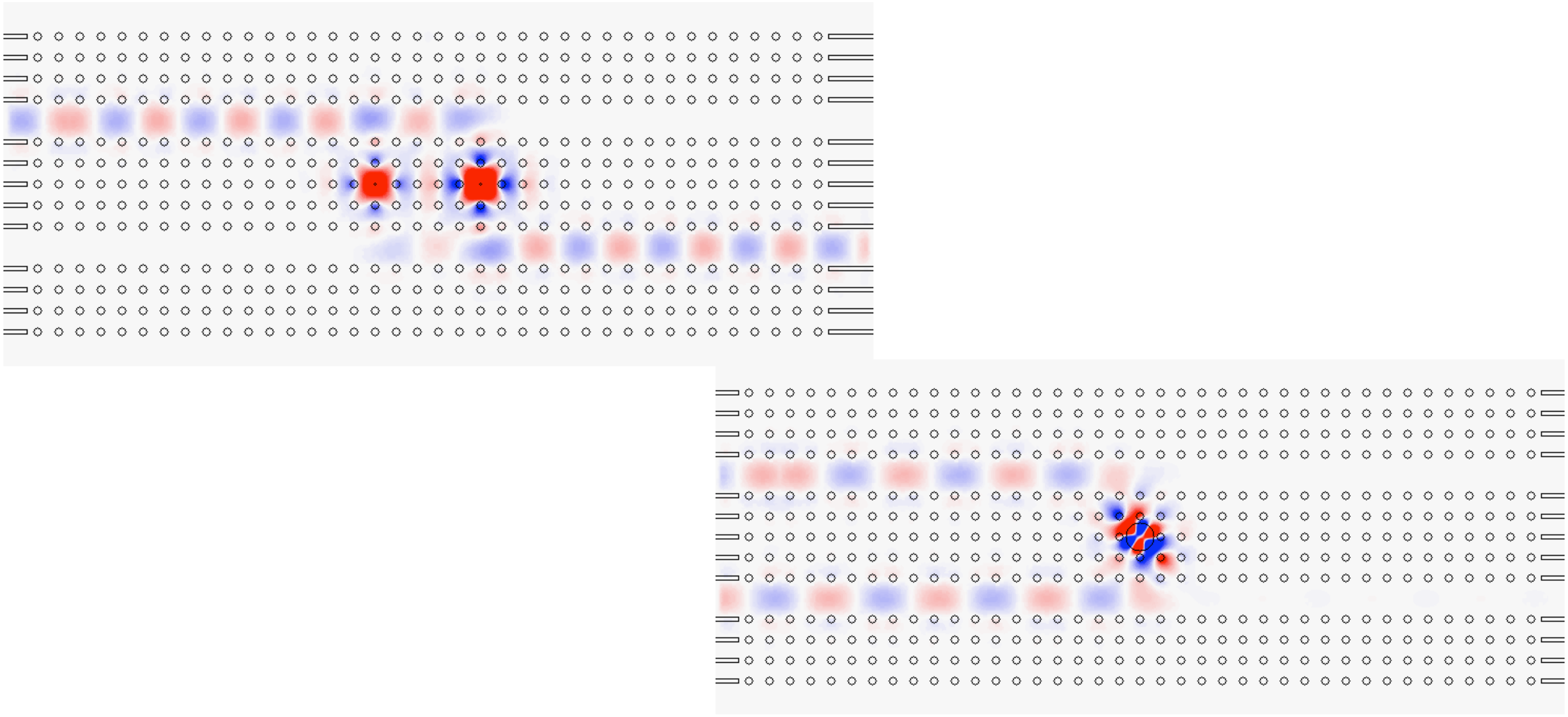
[S. Fan *et al.*, *Phys. Rev. Lett.* **80**, 960 (1998)]

Channel-Drop Filters



Criteria for Perfect Channel-dropping

- *Two resonant modes with even and odd symmetry*
- *Modes must be degenerate*
- *Modes must have equal decay rates*



Enough passive, linear devices…

Photonic crystal cavities:

tight confinement (~ $\lambda/2$ diameter)

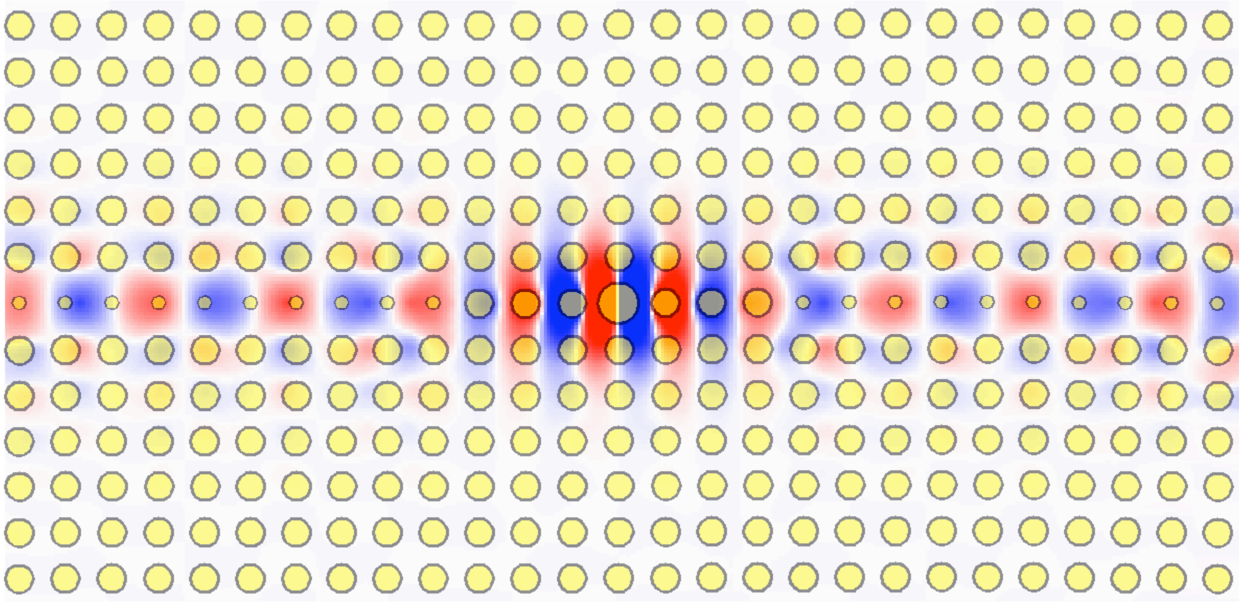
+ long lifetime (high Q independent of size)

= enhanced nonlinear effects

e.g. Kerr nonlinearity, Δn ~ intensity

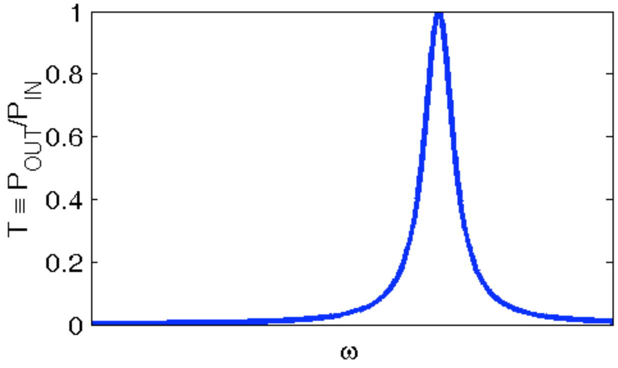
A ~~Linear~~ *Nonlinear* Filter

in →



→ out

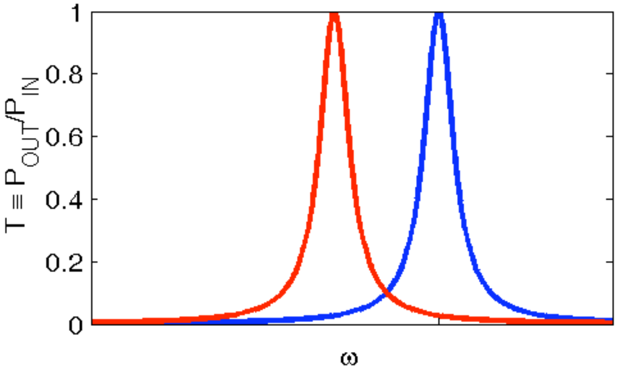
Linear response:
Lorenzian Transmisson



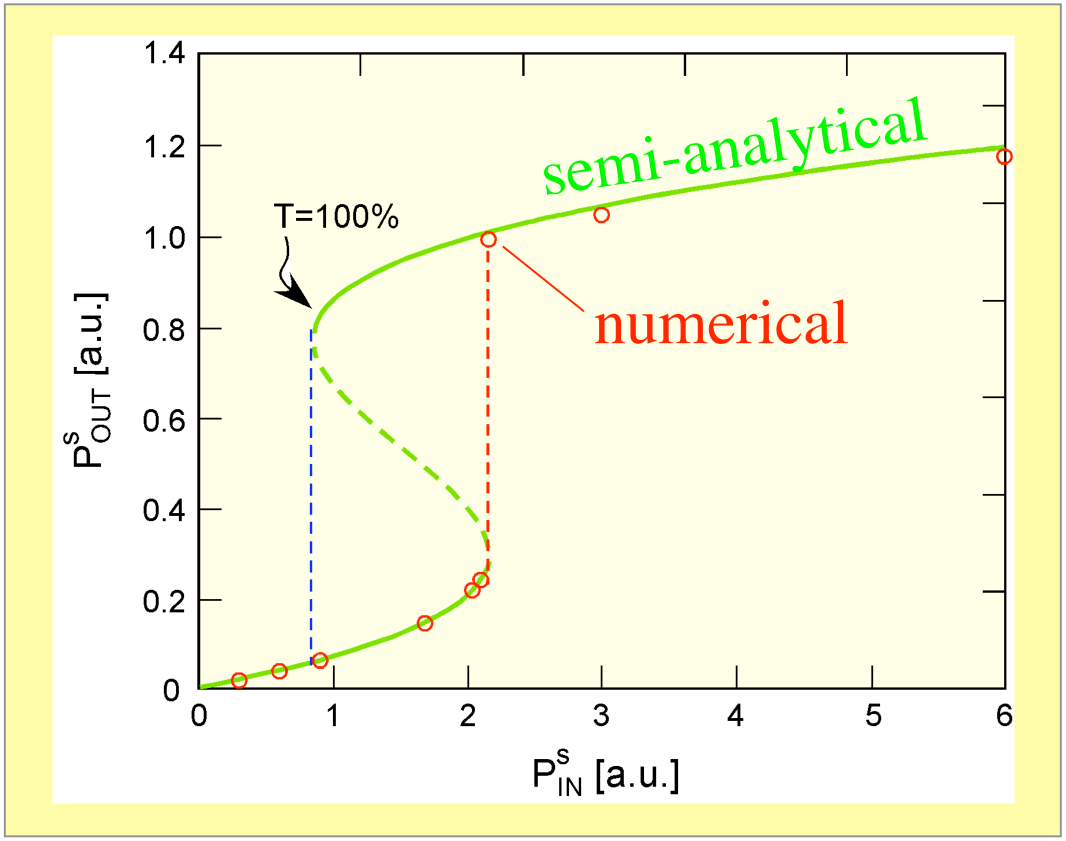
→

+ nonlinear
index shift

shifted peak



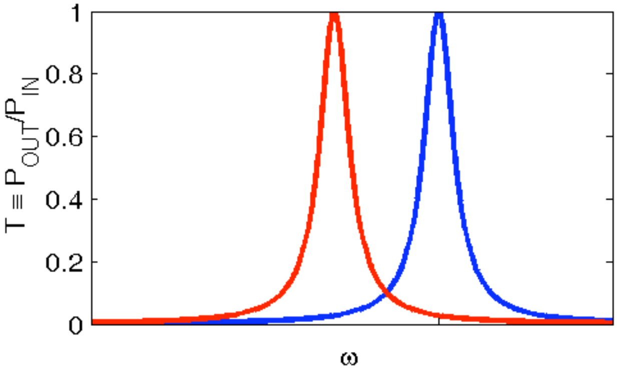
A ~~Linear~~ *Nonlinear* “Transistor”



Logic gates, switching, rectifiers, amplifiers, isolators, …

+ feedback

shifted peak



Bistable (hysteresis) response

Power threshold is near optimal
(~mW for Si and telecom bandwidth)

Enough passive, linear devices…

Photonic crystal cavities:

tight confinement (~ $\lambda/2$ diameter)

+ long lifetime (high *Q* independent of size)

= enhanced nonlinear effects

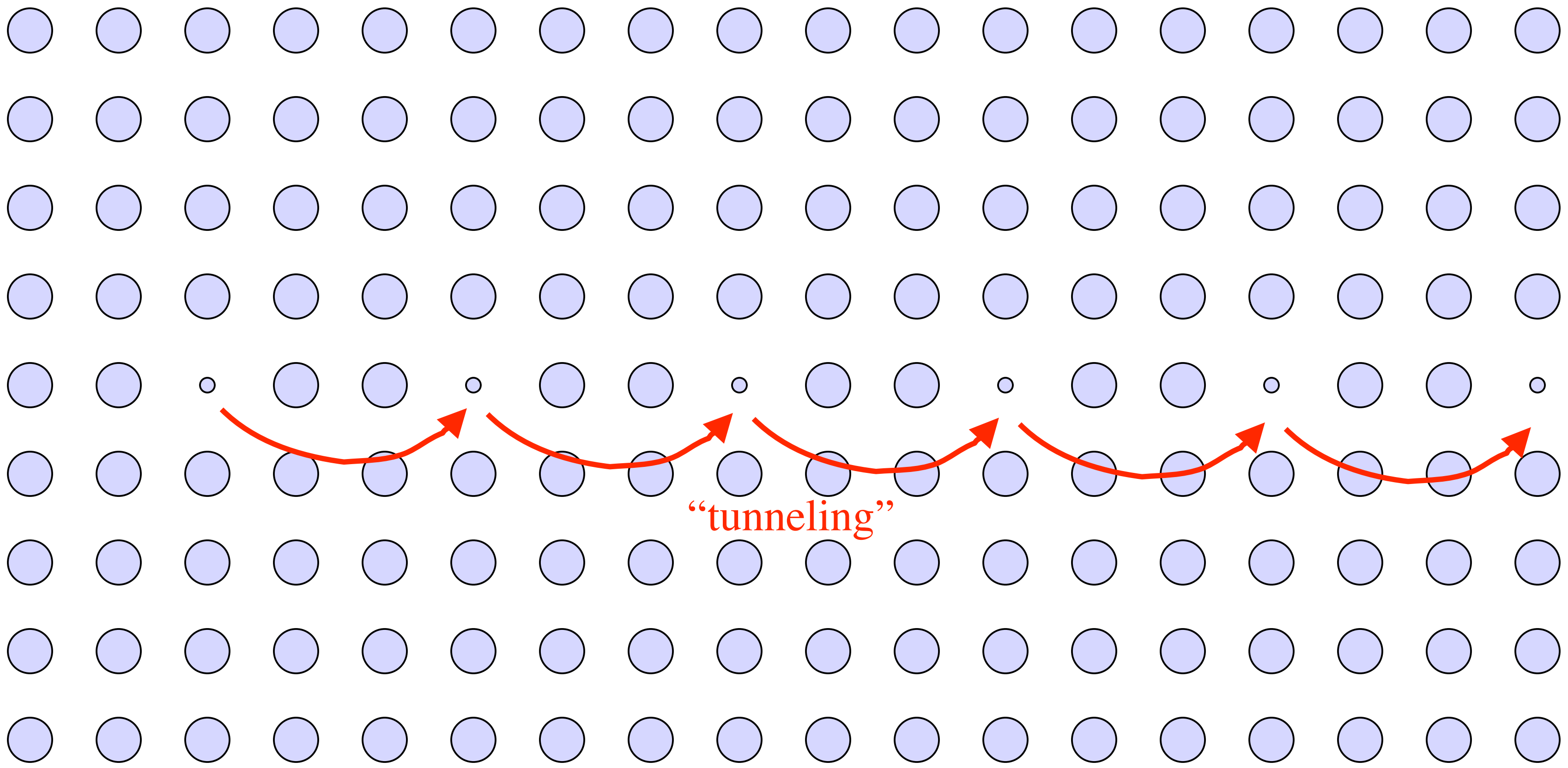
Photonic crystal waveguides:

tight confinement (~ $\lambda/2$ diameter)

+ slow light (e.g. near band edge)

= enhanced nonlinear effects

Cavities + Cavities = Waveguide

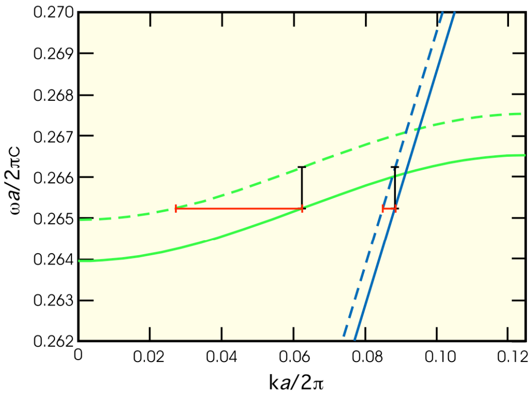


coupled-cavity waveguide (CCW/CROW): slow light + zero dispersion

[A. Yariv *et al*., *Opt*. *Lett*. **24**, 711 (1999)]

Enhancing tunability with slow light

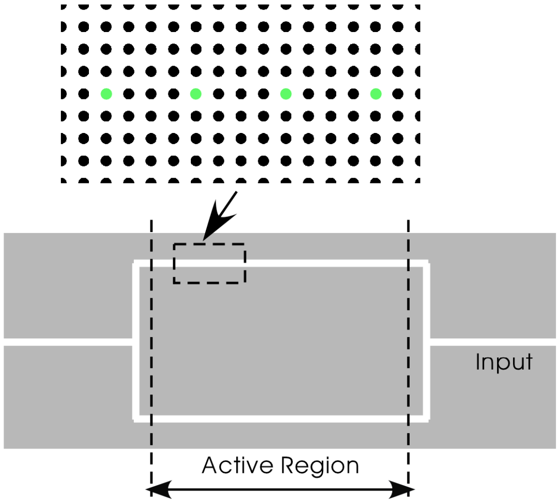
Photonic Crystal Slow-Light Enhancement of Non-linear Phase Sensitivity

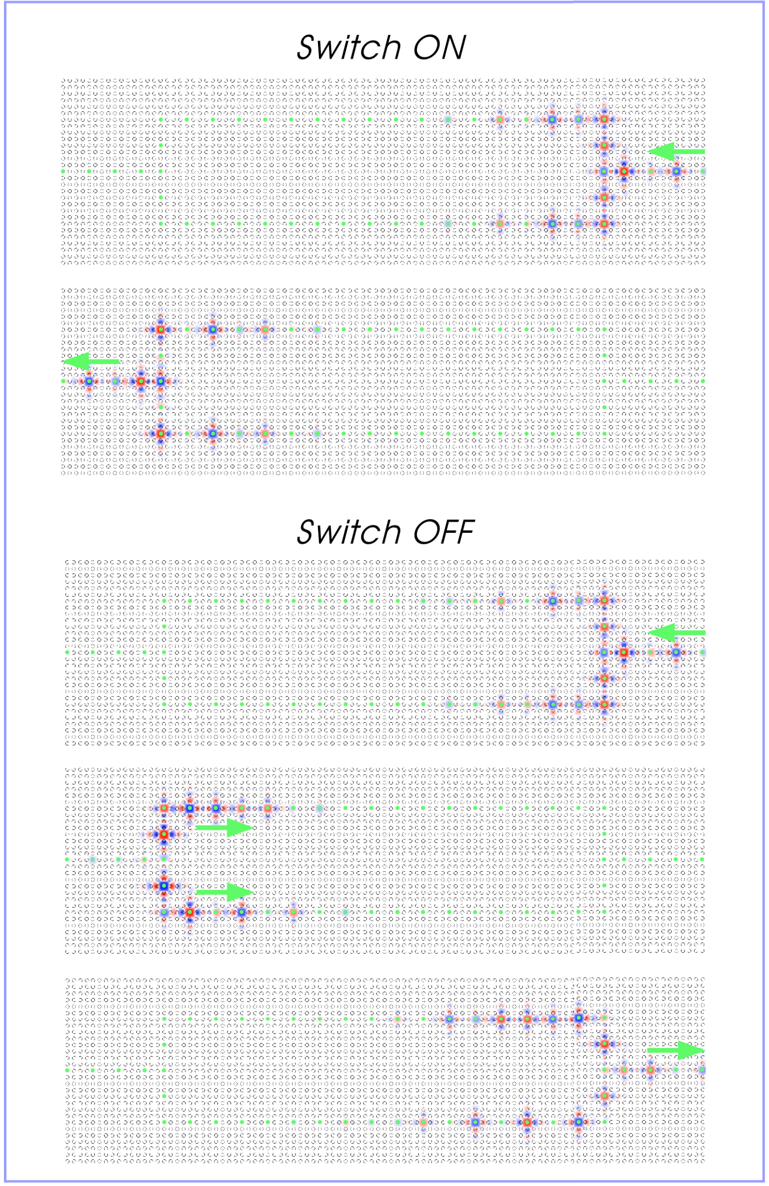


$$\delta\phi \sim \frac{\delta n}{n}\frac{\omega L}{v_g}$$

$$\frac{L}{\lambda_{air}} \sim \frac{n}{\delta n}\frac{v_g}{c}$$

Photonic Crystal Mach Zehnder Switch

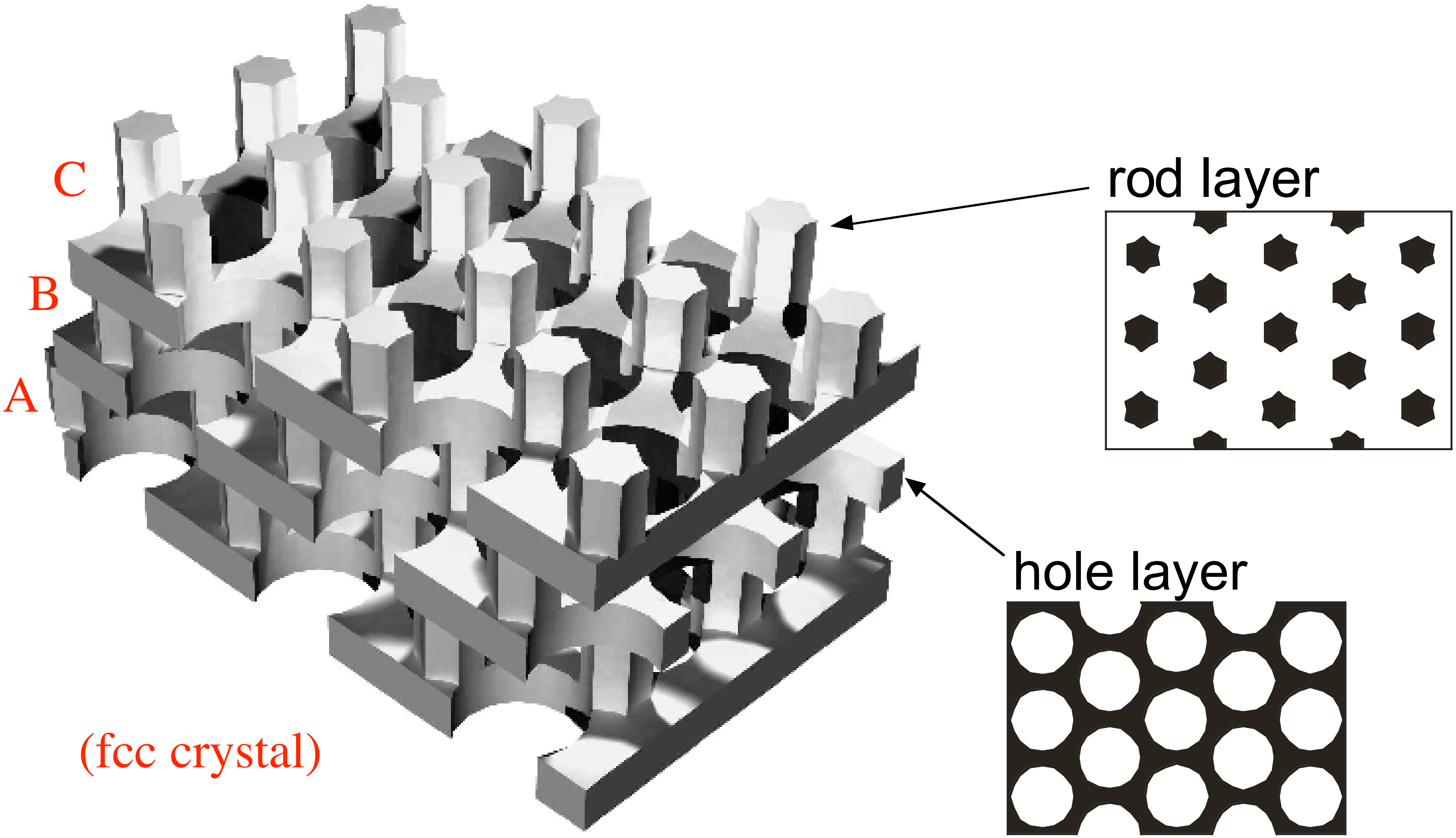




[M. Soljacic *et al.*, *J. Opt. Soc. Am. B* **19**, 2052 (2002)] periodicity:

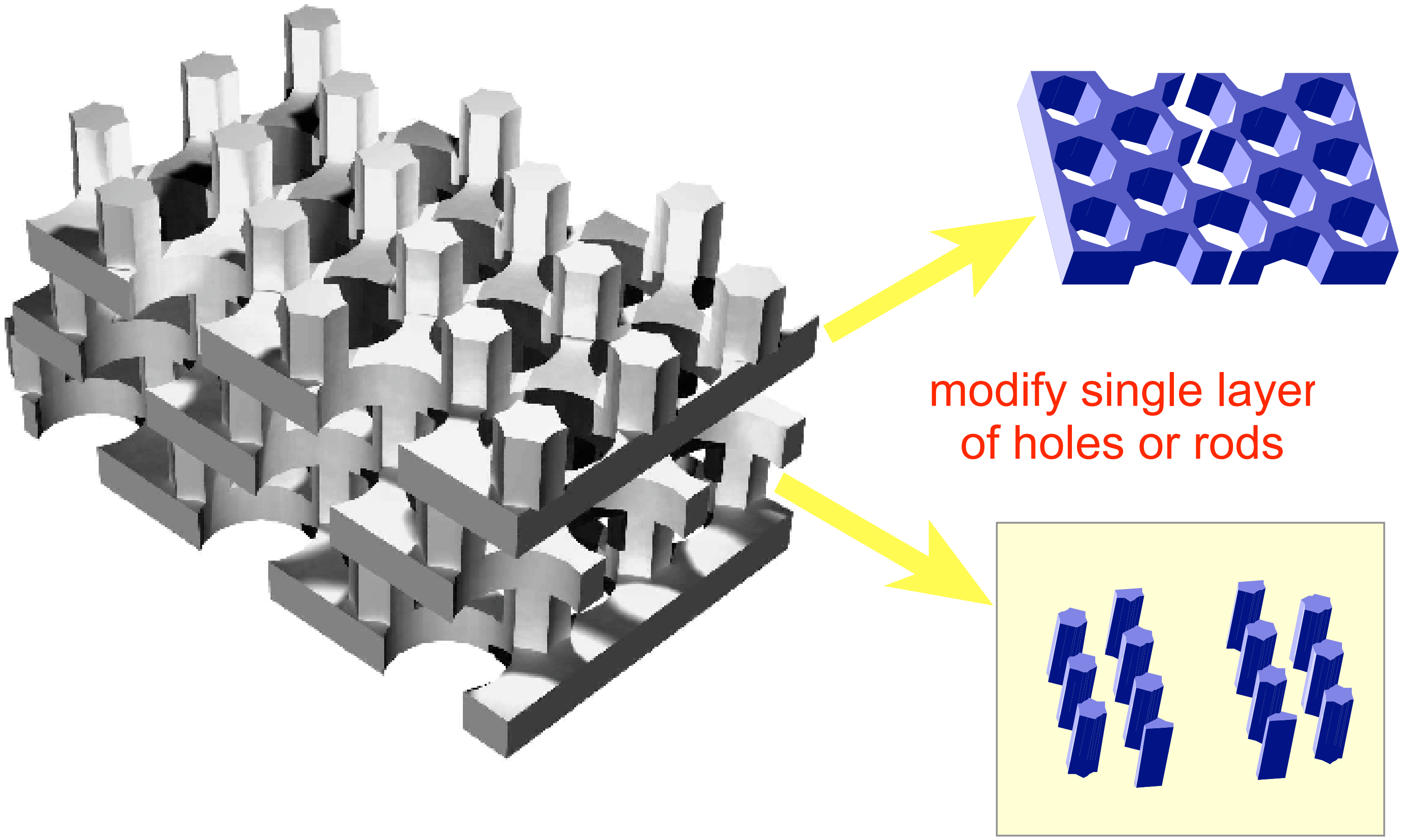
light is slowed, but not reflected

Uh oh, we live in 3d…



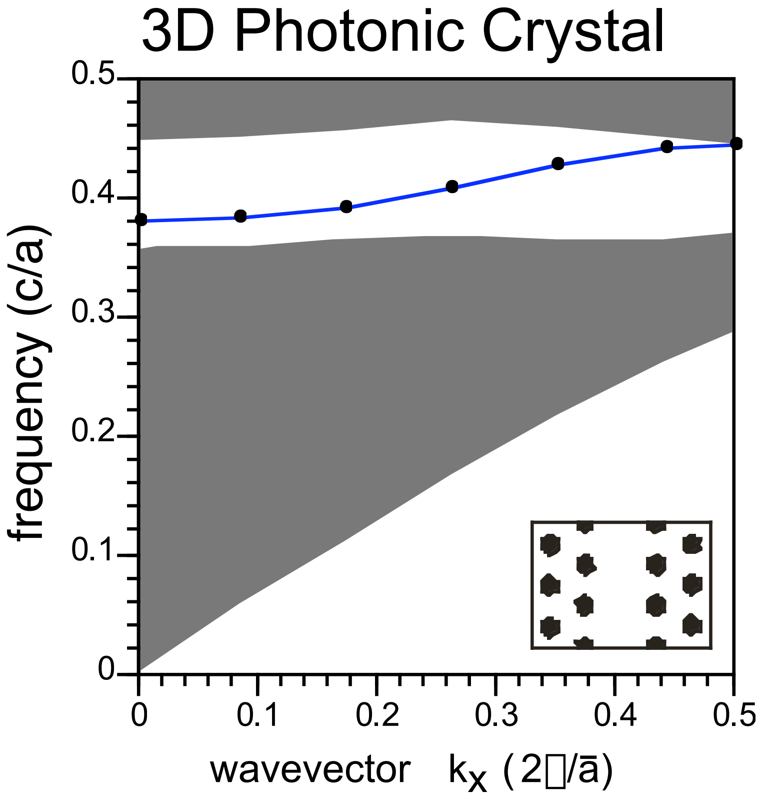
2d-like defects in 3d

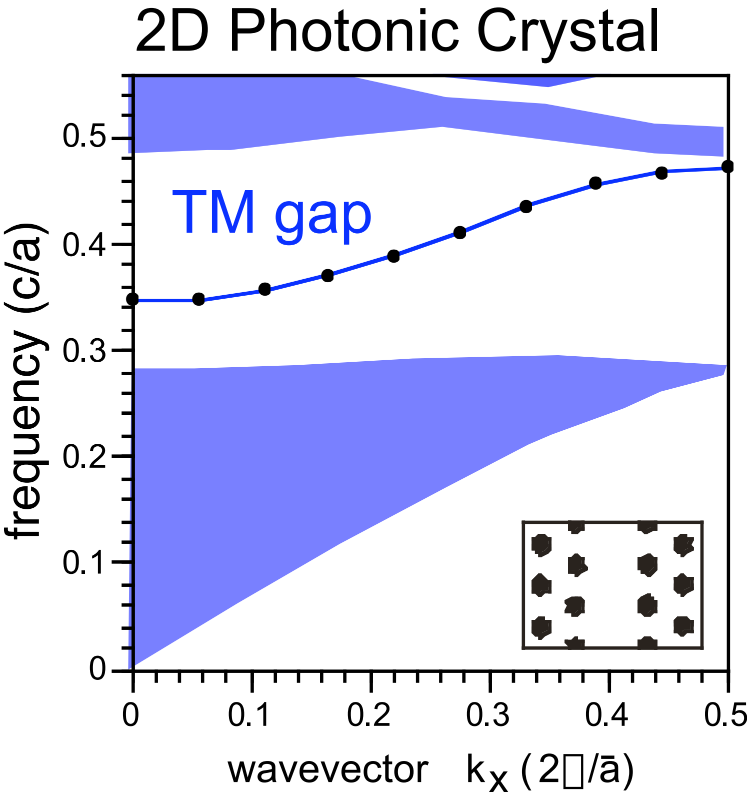
[M. L. Povinelli *et al.*, *Phys. Rev. B* **64**, 075313 (2001)]



modify single layer
of holes or rods

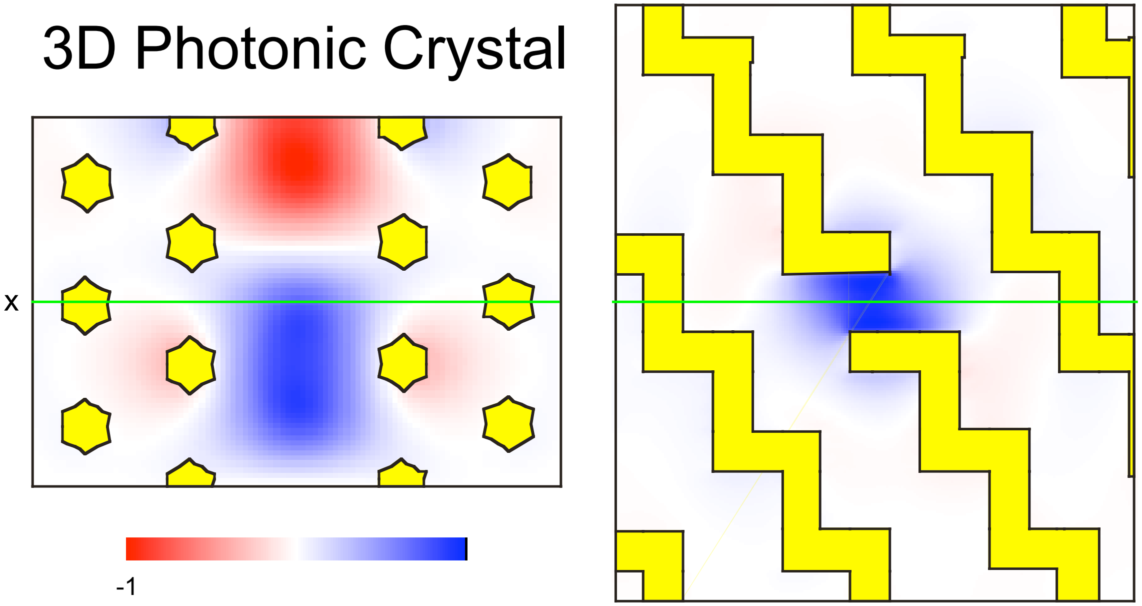
3d projected band diagram


3D Photonic Crystal

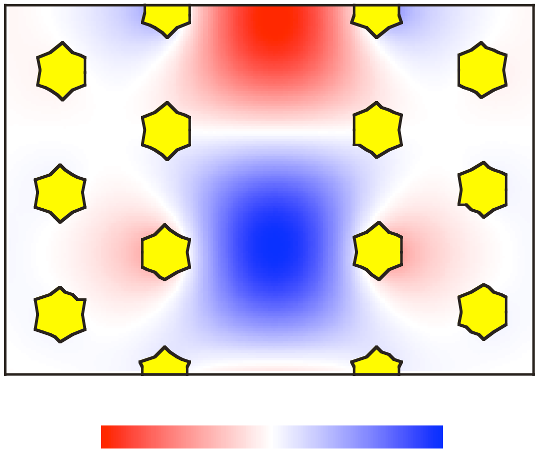

2D Photonic Crystal

2d-like waveguide mode

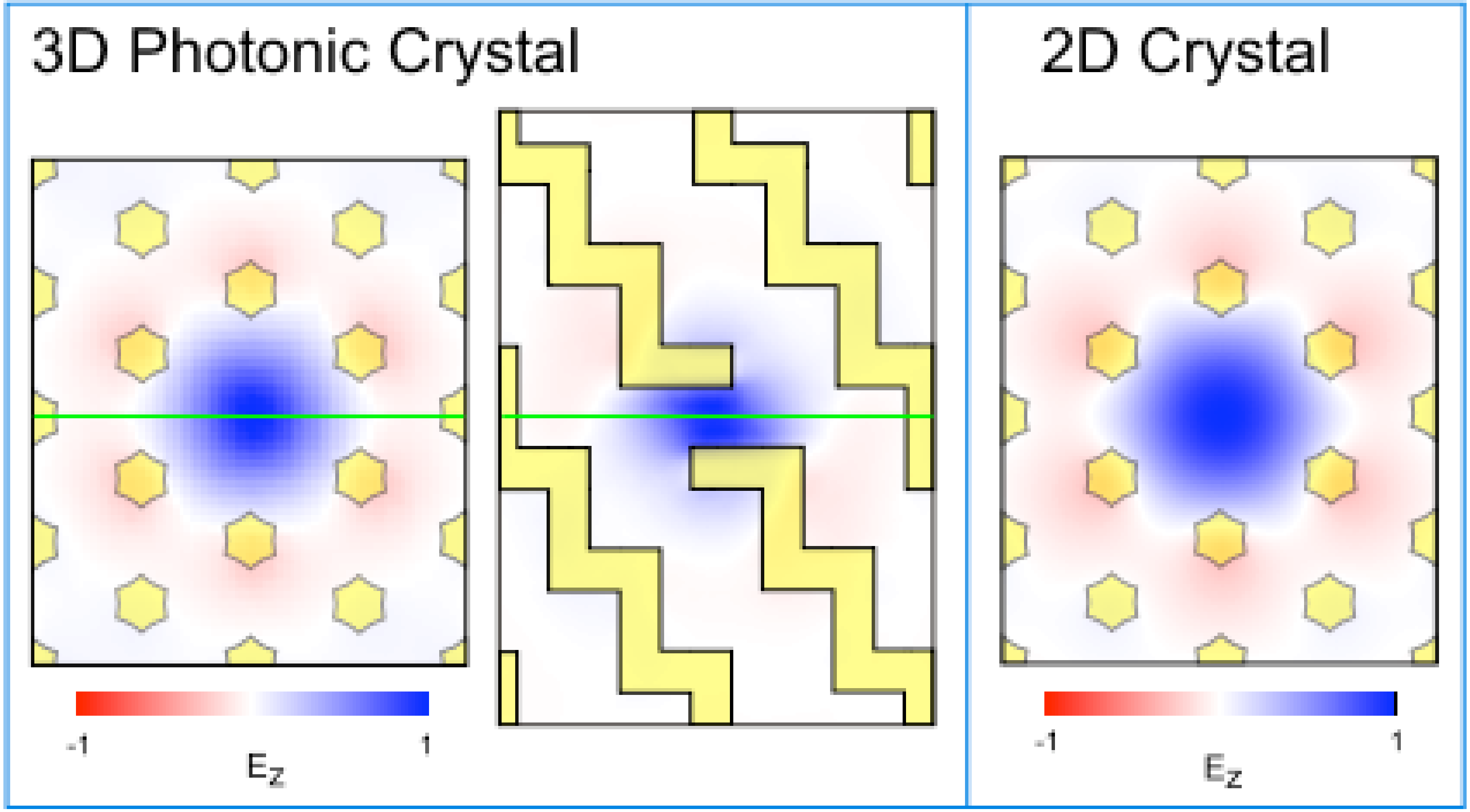
3D Photonic Crystal



2D Photonic Crystal



2d-like cavity mode



The Upshot

To design an interesting device, you need only:

symmetry

+ single-mode (usually)

+ resonance

+ (ideally) a band gap to forbid losses

Oh, and a full Maxwell simulator to get Q parameters, *etcetera*.