# Tailoring repulsive optical forces in nanophotonic waveguides

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We describe a mechanism and propose design strategies to selectively tailor repulsive–gradient-optical forces between parallel, nanophotonic waveguides via morphology augmented by slow-light band-edge modes. We show that at small separation lengths, the repulsive force can be made nearly 2 orders of magnitude larger than that of standard dielectric waveguides with a square cross section. The increased coupling interactions should enable a wider dynamic range of optomechanical functionality for potential applications in sensing, switching, and nanoelectromechanical systems.   © 2011 Optical Society of America

  *OCIS codes:*   130.5296, 160.5293.

Optical gradient forces are becoming an increasingly important area of research in nanophotonics with a number of emerging applications in sensing, switching, and nanoelectromechanical systems [1]. However, given the weak nature of such forces, experimental observation (usually through an indirect measurement of the mechanical–vibrational modes and transmission measurements [2,3]) requires delicate instrumentation with high sensitivity. As a result, the prospect of future nanotechnologies based on this phenomenon may be limited unless these forces can be significantly enhanced. Previous attempts to boost these forces have mainly relied on structures where the intensity of the electromagnetic field can be greatly increased, first through resonance [4–7] and more recently via slow-light modes [8]. Yet these prior proposals have little effect on the repulsive force, which has remained weak relative to the attractive force [2], thereby restricting the full range of dynamic functionality necessary in complex optomechanical systems. Here we demonstrate 2 orders of magnitude enhancement (potentially more, with no theoretical constraint on the upper bound) of the repulsive force between parallel dielectric waveguides through the careful design of waveguide morphology augmented by slow-light band-edge modes. We provide an intuitive explanation for the key role of geometrical structure on the repulsive force and propose design strategies for enhancing such forces. We then show that at small separation between the two waveguides and fixed input power, the optical gradient force can be made arbitrarily large via a reduction in the group velocity of the guided modes.

The optical gradient force arises from the evanescent coupling of the modes of two adjacent structures and is transverse to the light propagation direction. Povinelli *et al.* [9] showed that for the case of parallel-dielectric waveguides, the gradient force can be tuned to be either attractive or repulsive depending on the relative phase of the two guided modes and can be computed using the following analytic expression:

$$F = -\frac{1}{\omega}\frac{d\omega}{ds}\bigg|_{\mathbf{k}} U, \qquad (1)$$

where $\omega$ is the eigenmode frequency of the coupled-waveguide system, $s$ is the separation distance between the parallel waveguides, $k$ is the conserved wave vector, and $U$ is the total energy of the electromagnetic fields [9]. By convention, negative and positive values correspond to attractive and repulsive forces, respectively. Rakich *et al.* [10] later showed that Eq. (1) is mathematically equivalent to the Maxwell stress tensor method but has the advantage that it offers both physical insights into the underlying mechanisms of the gradient force and is simpler to compute. Equation (1) indicates that the magnitude of the force is proportional to the mode energy (or field intensity), a fact which has hitherto been exploited in various resonant [4–7] and slow-light [8] structures. Yet Eq. (1) also indicates that the sign (as well as the magnitude) of the force is determined by the rate of change of eigenfrequency with separation distance. This therefore suggests that in order to produce repulsive forces, the system must continually *increase* its eigenfrequency as the individual waveguides are brought in closer proximity (for an attractive force, the opposite is true: the system must *lower* its eigenfrequency). This latter point has not been explored in detail, and in this Letter we demonstrate the role of waveguide morphology on tailoring modal interaction to control the force characteristics.

Let us inspect more closely the connection between the mode profile and the resulting optical force for the original case of parallel Si ($n = 3.45$) waveguides with square cross section as shown in Fig. 1. This is a plot of the dimensionless force per unit length and input power ($F/L$) ($ac/P$) as a function of the dimensionless waveguide separation $s/a$, where $a$ is the square width, $L$ is the waveguide length, $c$ is the speed of light, and $P = v_g U/L$ is the total power transmitted through the coupled waveguides ($U$ is defined as before and $v_g$ is the group velocity). The waveguide axis is along $x$ and the separation is along $z$. In this Letter, we consider only $y$-odd modes such that the $E_x$, $E_y$, and $E_z$ fields are odd, even, and odd, respectively. Eigenmode computations of the parallel-waveguide system were performed by conjugate-gradient minimization of the Rayleigh quotient using a freely available plane wave solver [11]. The symmetric ($z$-even) mode with data points as solid-red squares shows a monotonically increasing attractive force ($F < 0$) with decreasing separation distance. The corresponding mode profiles at $s/a$ separation distances of 0
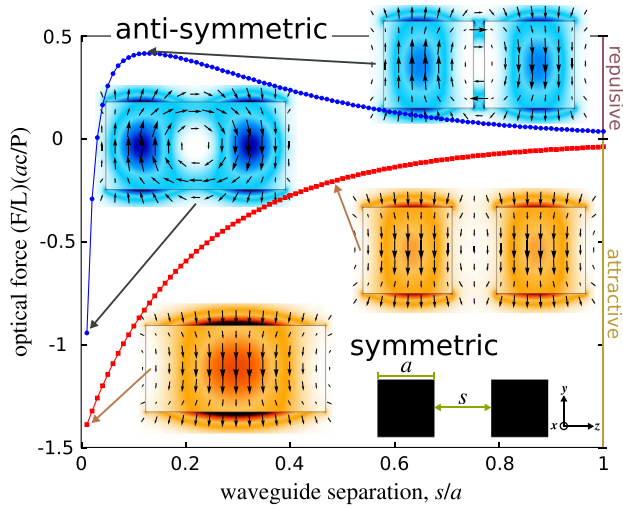
Fig. 1.   (Color online) Normalized force per unit length and input power versus waveguide separation for the symmetric ($z$-even, red, lower curve) and antisymmetric ($z$-odd, blue, upper curve) $y$-odd modes of parallel waveguides with a square profile (width $a$) at a fixed-axial wave vector of $\pi/a$. Insets show the in-plane **E**-field vector distribution (foreground) with the total **E**-field intensity in the background (darker is more intense).

and 0.5 show the in-plane **E**-field as the foreground and the total **E**-field intensity as the background. As the two waveguides come closer together and the interaction increases, the mode for the combined structure becomes localized between the waveguides along their axis of separation. This feature can be readily understood from the variational theorem [12] as the lowest-frequency eigenmode corresponds to the field pattern obtained from

$$\frac{\omega^2}{c^2} = \min_{\mathbf{E}} \frac{\int \mathrm{d}^3\mathbf{r} |\nabla \times \mathbf{E}(\mathbf{r})|^2}{\int \mathrm{d}^3\mathbf{r}\,\varepsilon(\mathbf{r}) |\mathbf{E}(\mathbf{r})|^2}. \qquad (2)$$

Equation (2) indicates that the fundamental mode will concentrate its **E**-field in regions of high-dielectric constant while minimizing the amount of spatial oscillations (preference is given to the former because it has inverse scaling). The symmetric mode's in-plane **E**-field components are mainly $E_y$, which, due to the continuity of this field component across a dielectric interface oriented in the same direction, implies that it can localize more and more inside the high-dielectric region as the waveguides are brought closer into contact (an antinodal line at the center of the combined-waveguide geometry ensures this). This explains how the mode progressively *lowers* its eigenfrequency with decreasing separation and therefore gives rise to a monotonically increasing attractive force for this symmetry profile. (The effect of $E_x$ is similar to $E_y$ because it transforms under $z$-mirror symmetry in the same way and is always parallel to the dielectric boundaries.) The antisymmetric ($z$-odd) mode, with data points as blue circles, shows a slowly increasing repulsive force with decreasing separation until a maximum is reached at approximately $0.13s/a$ before the onset of attractive forces. An inspection of the in-plane **E**-field distribution (shown as blue-shaded insets) reveals that $E_y$ is dominant outside the air gap where $E_z$ prevails. The odd–mirror-symmetry nodal line forces the $E_y$ field to zero in the center (the reason why $E_z$ predominates in

this area), but this must be done gradually to minimize the numerator of Eq. (2), which "pushes" apart the individual-waveguide modes (hence the initial onset of repulsion) while trying to remain as much inside the high-dielectric region as possible). Moreover, the inner vertical edges of the square profile also limit the spatial extent of air that the $E_z$ fields have to traverse as the mode coupling increases. This means that when the two waveguides are in contact and there is no air gap separating them, the **E**-field can reside mostly in the high-dielectric region. Taken together, both the relatively large cross-sectional area [minimizing the numerator of Eq. (2)] and the absence of an air gap between the structures when touching [maximizing the denominator of Eq. (2)] are the primary reasons why the system with a square profile can, beyond a certain threshold, continue to lower its eigenfrequency, and therefore no large repulsive force is obtained. What, then, happens to the repulsive force if we consider a different waveguide morphology?

Figure 2 is another plot of the optical force versus waveguide separation, this time for two different cross sections: a circle and hemicircle. While in both cases the symmetric mode shows a similar trend of increasing attractive force with decreasing separation as the square case, the repulsive-force behavior of the antisymmetric mode, particularly for the hemicircle, is now quite different—increasing monotonically with decreasing separation. As the waveguides approach each other, the distinct waveguide modes will once again tend to keep apart so as to minimize the rate of field oscillations. And this is where the hemicircle with its flat outer edges and reduced cross-sectional area relative to the circle leads to a larger amount of **E**-field to leak out of the high-dielectric region. Furthermore, unlike the square case, the convex inner surface causes more of the fields to spread out into the air as mode coupling increases. The eigenfrequency, as a result, can only increase as the waveguides are brought closer together, which therefore
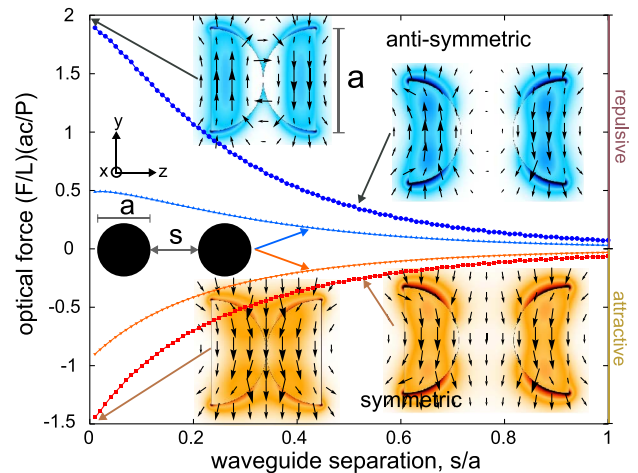


Fig. 2.   (Color online) Normalized force versus waveguide separation for the symmetric ($z$-even, red) and antisymmetric ($z$-odd, blue) $y$-odd modes of parallel waveguides with circular and hemicircular profile (diameter $a$) at a fixed-axial wave vector of $\pi/a$. Insets show the in-plane **E**-field vector (foreground) with the total **E**-field intensity in background (darker is more intense).
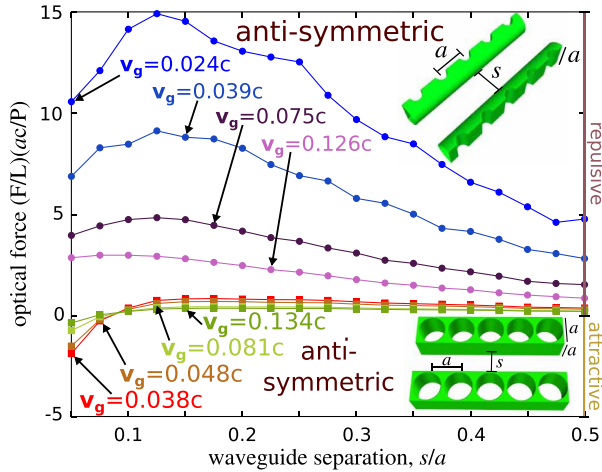
Fig. 3.   (Color online) Normalized force versus waveguide separation of four antisymmetric slow-light modes for two parallel waveguide systems, one with hemicircular (solid circles) and the other with square (solid squares) cross sections, both with air holes (periodicity $a$, radius $0.2a$) along the waveguide axis.

gives rise to increasing repulsion. And so the waveguide morphology, through a combination of a reduced cross-sectional area and a surface profile that squeezes the fields into the air region as coupling increases, plays a critical role in determining the characteristics of the repulsive force.

To obtain even larger repulsive forces, we now propose a 3D waveguide structure that combines the morphology necessary for repulsion at small separation lengths with slow-light modes that boost the magnitude of the force by increasing the field energy. A previous proposal used the slow-light effect to obtain large-attractive forces in a waveguide-substrate system but did not consider the repulsive force [8]. The photonic-crystal (PC) waveguide consists of a lattice of air holes (periodicity $a$, radius $0.2a$) in Si with a hemicircular cross section (diameter $a$) as shown in the inset schematic of Fig. 3 (a circular cross section also shows an enhanced repulsive force, though not as large as the hemicircle case). The dispersion relation for the $y$-odd fundamental mode of this slow-light waveguide geometry has a band-edge frequency at $0.3a/\lambda$ where the group velocity of a propagating mode is zero. As the mode energy [$U$ in Eq. (1)] is inversely proportional to the group velocity, the force therefore can be made arbitrarily large for an ideal (i.e., no material absorption, nonlinearities, and surface roughness) slow-light waveguide depending on how close the operating mode is chosen to the band edge [8]. We investigate the optical forces of a set of four different slow-light modes with decreasing group velocity (although the theoretical lower bound is zero, coupling to such modes from an external source becomes more and more challenging due to the inherent impedance mismatch between the modes). The antisymmetric modes of the hemicircular waveguide in Fig. 3 are nonmonotonic (principally due to the larger degrees of freedom in the

3D unit cell) with the repulsive peak increasing as the group velocity is lowered. Also shown in Fig. 3 are the antisymmetric modes of an identical waveguide with a square cross section, which has negligible repulsive force *even with* the slow-light effect. The symmetric modes for both structures meanwhile show the same trends as before and are not shown. For comparison, the maximum repulsive force for the lowest–group-velocity ($0.024c$) antisymmetric mode of the hemicircular PC waveguide is nearly 15 at a separation of $0.125s/a$, whereas the square cross-section structure used in [9] (mode group velocity of $0.025c$) is $0.53$ at a separation of $0.2s/a$. Our proposed waveguide geometry thus represents an enhancement of the repulsive force by a factor of nearly 30 using this particular slow-light mode (as a further demonstration, a slow-light mode with group velocity of $0.0046c$ leads to an enhancement by more than 2 orders of magnitude).

In summary, we have demonstrated general design principles to markedly enhance repulsive optical forces between parallel waveguides by the proper choice of waveguide morphology augmented by slow-light effects. This approach can now be used to further boost the forces via morphology optimization to determine the ideal geometry for a specific material system and should enable progress toward the ultimate goal of dynamic, reconfigurable optomechanical devices.

†These authors contributed equally to this work.

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